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Nuts in BFO's Nutshell:

Revisions to the Bi-categorial Axiomatization of BFO

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Prefatory Comments

This report is published for the record. Section 1 (page 2) is an amended version of section 4 of BFO in a Nutshell (IFOMIS Report, 06/03). Section 2 (Page 21) is an extended version which explicits the temporally qualified vocabulary for SNAP alluded to until then. This last version forms the basis of the integration of BFO in the library of ontologies for the WonderWeb project (Deliverable 18).

The main difference between what is given here and the axiomatization presented in the WonderWeb stuff is that here both temporally qualified and unqualified vocabulary are defined on the whole domain, whereas the library of ontology version defines temporalized vocabulary on SNAP and non temporalized on SPAN and introduces ad hoc temporalized relations on SNAP-SPAN.

There are still known problems with the axiomatization. In particular, the mereotopology misses some definitions and is not well tidy up. These revisions were almost exclusively concerned with temporal issues.

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Reference

Grenon, P. BFO in a Nutshell: A Bi-categorial Axiomatization of BFO and Comparison with DOLCE, IFOMIS Report, 06/03, 2003.

WonderWeb, Deliverable 18:
http://wonderweb.semanticweb.org/deliverables/D18.shtml

1 Formal Characterization of BFO, revised 21 12 03

Pierre Grenon, IFOMIS (pierre.grenon@ifomis.uni-leipzig.de)

4 Formal characterization of BFO

The language used here is a first-order one with equality. It contains the usual logical symbols (infix notation): $\sim$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, $\rightarrow$ for material implication, $\leftrightarrow$ for logical equivalence, $=$ for equality, $\forall$ (respectively $\exists$) for the universal (respectively existential) quantifier. I use the following conventions:

– variables (individuals): x, y, z, v, w, …

– constants (individuals): lower case strings of Latin letters

– predicates: concatenations of capitalized strings of Latin letters (prefix notation)

– In a couple of occurrences I will use schematic letters.

– I indicate tentative uses of the symbols 'PT', 'SB', and 'DJ' from (Masolo *et al.*, 2003) for partition, subsumption, and disjointness among categories (with the expected interpretations given the earlier discussions on universals). However, I also provide alternative formalization simulating subsumption and related notions via material implication (this comes very close to BFO's approach in terms of universals, one only needs to replace monadic predication with the corresponding statement using instantiation).

I do not offer a full logic, in particular there will be no consideration on a deductive system. I simply assume all the tautologies of classical predicate calculus and standard rules of inference, the alleged corollaries or theorems are so in natural deduction. All formulas are given a number by order of appearance. The number should not be given any specific signification (although, of course, new formulas are asserted in contexts which generally assume previously asserted formulas or introduction of the relevant primitives). Furthermore, 'A' next to a number indicates that the formula is an axiom, 'D' that it is a definition, 'C' that it is a putative corollary.

There is a disclaimer on the axiomatization provided here. The sources – when there is any – are somewhat heterogeneous, even in a single section (mereotopology in particular). Starting with section 4.5, I also had to draw on programmatic and sometimes merely suggested material or make new suggestions. Obviously, there must be flows and this material should be regarded as a transitory result of a work in progress.

The axiomatization is presented modularly in the spirit of BFO. In particular, SNAP is first axiomatized in a self-standing manner. Later on, a temporalized variant of this vocabulary is introduced. This allows to simulate only some of the features of the meta-ontological variant of BFO's formalism.

4.1 Mereology

Material in this section is based on or adapted from (Simons, 1987; Smith, 1997; Smith and Varzi, 2000)

Primitive term

Part(x,y) means that x is a part of y. Part is the only primitive mereological relation.

Defined terms

ProperPart(x,y) means that x is a proper part of y, i.e., x and y are distinct but x is a part of y (or a is part of y but b is not part of x).

Overlaps(x,y) means that x overlaps with y, i.e., x and y have a part in common.

Underlaps(x,y) means that x and y are both parts of a third entity.

Fusion(y,[Φx]) means that y is the fusion of the Φers, where Φ is a formula of the language with at least one free variable. I do not use an operator for definite descriptions. Rather, I systematically use relations. The definition and further uses are probably best read as axiom schemas. Uniqueness of the fusion is demanded in the definition. Existential axioms need be supplied contextually.

Sum(x,y,z) means that x is the sum of y and z, i.e., x is the fusion of all entities overlapping y or x.

Difference(x,y,z) means that x is the difference between y and z, i.e., the fusion of all parts of y which do not overlap z.

Product(x,y,z) means that x is the product of z, i.e., the fusion of all common parts of y and z.

Complement(x,y) holds when x is the complement of y, i.e., the sum of all entities which do not overlap with y. In practice this will be useless unless relative to a given category. An amended concept could be introduced by requiring the fused entities to instantiate a given universal, for instance, a cumulative one instantiated by y.

Cumulative universals (predicates) are such that the sum of their instances falls under them as well.

Dissective ones are such the parts of their instances is one of their instances as well.

Axioms for Part

$\forall x\ Part(x,x)$ (A 1)

$\forall x \forall y \forall z\ ((Part(x,y) \wedge Part(y,z)) \rightarrow Part(x,z))$ (A 2)

$\forall x \forall y\ ((Part(x,y) \wedge Part(y,x)) \rightarrow x = y)$ (A 3)

Definitions

$ProperPart(x,y) \equiv_{def} Part(x,y) \wedge \sim(x = y))$ (D 4)

$Overlaps(x,y) \equiv_{def} \exists z\ (Part(z,x) \wedge Part(z,y))$ (D 5)

$\text{Underlaps}(x,y) \equiv_{def} \exists z\ (\text{Part}(x,z) \land \text{Part}(y,z))$ (D 6)

$\text{Fusion}(y,x[\Phi x]) \equiv_{def} \forall z\ ((\text{Overlaps}(z,y) \leftrightarrow \exists x\ (\Phi x \land \text{Overlaps}(z,w)))$

$\land\ \forall z'\ (\text{Overlaps}(z,y) \leftrightarrow \exists x\ (\Phi x \land \text{Overlaps}(z,w))) \rightarrow z = z'))$ (D 7)

$\text{Sum}(x,y,z) \equiv_{def} \text{Fusion}(x,w[\text{Part}(w,y) \lor \text{Part}(w,z)])$ (D 8)

$\text{Difference}(x,y,z) \equiv_{def} \text{Fusion}(x,z[P(z,x) \land \sim\text{Overlaps}(z,y)])$ (D 9)

$\text{Product}(x,y,z) \equiv_{def} \text{Fusion}(x,w[\text{Part}(w,y) \land \text{Part}(w,z)])$ (D 10)

$\text{Complement}(x,y) \equiv_{def} \text{Fusion}(x,z[\sim\text{Overlaps}(z,y)])$ (D 11)

Two axiom schemas

Using the schematic letter '**P**' for a given predicate symbol, here are a couple of definitions adapted from (Simons, 1987):

(D 12) **P** is dissective:

$\forall x \forall y\ ((\mathbf{P}(x) \land \text{Part}(y,x)) \rightarrow \mathbf{P}(y))$ (D 12)

(D 13) **P** is cumulative:

$\forall x \forall y \forall z\ ((\text{Sum}(x,y,z) \land \mathbf{P}(x) \land \mathbf{P}(y)) \rightarrow \mathbf{P}(z))$ (D 13)

4.2 Mereotopology

Material in this section is based on or adapted from (Smith, 1998; Smith and Varzi, 2000)

Primitive terms

BoundaryFor(x,y) means that x is a bona fide boundary for y; x is not necessarily the whole boundary of y, but any part of it. (Contrast with BoundaryOf to be defined.) A bona fide boundary for an entity is to be understood as a partial external delineation of that entity. Boundaries are lower dimensional entities (e.g., a section of a sphere is a boundary for a ball; a section of a circle is one for a disk; a point for a line). Bona fide boundaries are not all parts of the entities they bound, this is the case for closed entities (it is definitional for them).

FiatBoundaryFor(x,y) means that x is a fiat boundary for y. FiatBoundaryFor is the fiat counterpart of BoundaryFor. These are parts of the entities they are fiat boundaries for. A fiat boundary is for instance the delineation between two component parts of an entity (they are typically regarded as the products of convention).

Defined terms

BoundaryOf(x,y) means that x is the complete (bona fide) boundary of y. The boundary of an entity is the fusion of all entities which are (bona fide) boundaries for this entity. The boundary of an entity is therefore a boundary for that entity.

Closure(x,y) means that x is the closure of y. The closure of an entity is the sum of this entity with its boundary.

Interior(x,y) means that x is the interior of y. The interior of an entity is the difference between this entity and its closure.

WeaklyConnected(x) means that x is weakly connected, i.e., x is such that any two entities it is the sum of are such that their closure overlap. This is (Smith and Varzi, 2000)'s Connected.

MildlyConnected(x) means that x is mildly connected, i.e., x is such that any two entities it is the sum of are such that one overlaps with the closure of the other or vice versa. This is (Smith and Varzi, 2000)'s Connected*.

StronglyConnected(x) means that x is strongly connected, i.e., its interior is mildly connected.

ConnectsWith(x,y) means that x is connected to y, i.e., x and y overlap or x overlaps with the closure of y or y overlaps with the closure of x.

ExternallyConnectsWith(x,y) means that x is connected to y but they do not overlap.

Closed(x) means that x is closed, i.e., it is its own closure. A bona fide boundary – in particular, the boundary of this entity – for closed entity is a part of this entity.

InternalPart(x,y) (resp. FiatInternalPart(x,y)) means that x is a part of y and no boundary for (resp. fiat boundary for) x overlaps with y.

Boundary(x) means that x is a boundary of an entity (at least one).

FiatInternalPart(x,y) means that x is a fiat part of y.

FiatBoundary(x) means that x is a fiat boundary of some entity.

FiatConnected(x) means that x is a fiat entity which is self-connected.

Axioms for (Fiat)BoundaryFor

$\forall x \forall y \forall z\ ((\text{Part}(x,y) \land \text{BoundaryFor}(y,z)) \rightarrow \text{BoundaryFor}(x,z))$ (A 14)

$\forall x \forall y\ (\text{FiatBoundaryFor}(x,y) \rightarrow \text{Part}(x,y))$ (A 15)

$\forall x \forall y \forall z\ ((\text{Part}(x,y) \land \text{FiatBoundaryFor}(y,z)) \rightarrow \text{FiatBoundaryFor}(x,z))$ (A 16)

Definitions

$\text{BoundaryOf}(x,y) \equiv_{def} \text{Fusion}(x,z[\text{BoundaryFor}(z,y)])$ (D 17)

$\text{Closure}(x,y) \equiv_{def} \forall z\ (\text{BoundaryOf}(z,x) \rightarrow \text{Sum}(x,y,z))$ (D 18)

$\text{Interior}(x,y) \equiv_{def} \forall z\ (\text{Closure}(z,y) \rightarrow \text{Difference}(x,y,z))$ (D 19)

$\text{WeaklyConnected}(x) \equiv_{def} \forall y \forall z \forall v \forall w\ ((\text{Sum}(x,y,z)$
$\land \text{Closure}(v,y) \land \text{Closure}(w,z)) \rightarrow \text{Overlaps}(v,w))$ (D 20)

$\text{MildlyConnected}(x) \equiv_{def} \forall y \forall z \forall v \forall w\ ((\text{Sum}(x,y,z)$
$\land \text{Closure}(v,y) \land \text{Closure}(w,z)) \rightarrow (\text{Overlaps}(v,z) \lor \text{Overlaps}(w,y))$ (D 21)

StronglyConnected(x) $\equiv_{def}$ ∀y (Interior(y,x) → MildyConnected(y)) (D 22)

ConnectedWith(x,y) $\equiv_{def}$ ∀v∀w ((Closure(v,x) ∧ Closure(w,y))
→ (Overlaps(x,y) ∨ Overlaps(v, y) ∨ Overlaps(x, w)) (D 23)

ExternallyConnectsWith(x,y)
$\equiv_{def}$ ConnectsWith(x,y) ∧ ~Overlaps(x, y) (D 24)

Closed(x) $\equiv_{def}$ Closure(x,x) (D 25)

InternalPart(x,y) $\equiv_{def}$ Part(x,y)
∧ ∀z (BoundaryFor(z,y) → ~Overlaps(x,z)) (D 26)

Boundary(x) $\equiv_{def}$ ∃y BoundaryFor(x,y) (D 27)

FiatInternalPart(x,y) $\equiv_{def}$ Part(x,y)
∧ ∀z (FiatBoundaryFor(z,y) → ~Overlaps(x,y)) (D 28)

FiatBoundary(x) $\equiv_{def}$ ∃y FiatBoundaryFor(x,y) (D 29)

Additional Axioms

∀x∀y (Closure(x,y) → Part(y,x)) (A 30)

∀x∀y ((Closure(x,y) ∧ Closure(z,x)) → Part(z,x)) (A 31)

∀x∀y∀z∀u∀v∀w∀z' (Sum(x, y, z) ∧ Closure(u,x)
∧ Closure(v,y) ∧ Closure(w,z) ∧ Sum(z',v,w)) → z = z') (A 32)

∀x∀y (BoundaryOf(x,y) → BoundaryFor(x,y)) (C 33)

∀x∀y ((Closed(x) ∧ BoundaryFor(x,y)) → Part(x,y)) (C 34)

∀x∀y∀z ((Sum(x,y,z) ∧ StronglyConnected(x))
→ FiatConnectedWith(y,z)) (A 35)

∀x ((Boundary(x) ∧ FiatConnected(x))
→ ∃y∃z (FiatConnected(y) ∧ BoundaryFor(x,y) ∧ InternalPart(z,y))) (A 36)

∀x ((Boundary(x) ∧ FiatConnected(x)) → ∃y∃z
(FiatConnected(y) ∧ FiatBoundaryFor(x,y) ∧ FiatInternalPart(z,y))) (A 37)

The following are held to be theorem by (Smith and Varzi, 2000)

∀x∀y∀z ((BoundaryFor(x,y) ∧ BoundaryFor(y,z))
→ BoundaryFor(x,z)) (C 38)

∀x∀y∀z ((BoundaryFor(x,y) ∧ Complement(z,y))
→ BoundaryFor(x,z)) (C 39)

∀x∀y ~(ExternallyConnectsWith(x,y) ∧ Closed(x) ∧ Closed(y)) (C 40)

∀x∀y∀z ((FiatBoundaryFor(x,y) ∧ FiatBoundaryFor(y,z))
→ FiatBoundaryFor(x,z)) (C 41)

4.3 Dependence

Material in this section is based on or adapted from (Smith, 1997; Smith, 1998)

Primitive term

SD(x,y) means that x is specifically dependent on y. Specific dependence is defined by (Smith, 1997) modally and SD(x,y) means that x and y do not overlap and x is such that it necessitates the existence of y in order to exist. Notice in particular that specific dependence is then not a form of parthood. Here, without a modal language, I am taking dependence as primitive.

Defined terms

MSD(x,y) means that x is specifically dependent on y and are distinct entity and y is specifically dependent on x.

OSD(x,y) means that x is specifically dependent on y but y is not dependent on a.

$\forall x \forall y\ (SD(x,y) \rightarrow {\sim}Overlaps(x,y))$ (A 42)

$MSD(x,y) \equiv_{def} SD(x,y) \wedge SD(y,x)$ (D 43)

$OSD(x,y) \equiv_{def} SD(x,y) \wedge {\sim}SD(y,x)$ (D 44)

Note : These are all relations among particulars.

4.4 Location

Actually, I will give no general theory of location or regions. This is to keep in line with the modular spirit of BFO and following (Grenon, 2003b). Each of SNAP and SPAN have their theories of location and their adequate primitive. Practically, here, I will only use exact location. A general theory of location can be extrapolated from Casati and Varzi's treatment, in particular their (1996) and (1999).
In the case of the static account of SNAP, location will be a binary relation, while in the case of its temporally sensitive treatment, a ternary one (location in space at a time).

4.5 BFO

In keeping with the modular framework of BFO, following (Grenon, 2003b) I take SNAP and SPAN entities as primitive notions. We can always introduce the term 'Entity' as applying to entities of any of the kinds used here. Since this is only a partial rendition of BFO (most importantly not including universals), I leave it open whether SpanEntity and SpanEntity form partitions of this putative Entity, i.e., whether there are any other kinds of entities. Here, the instances of SnapEntity and SpanEntity are all particulars.

SnapEntity(x) means that x is a SNAP entity.

SpanEntity(x) means that x is a SPAN entity.

DJ(SnapEntity,SpanEntity) (D 45)

$\forall$x ~(SnapEntity(x) $\wedge$ SpanEntity(x)) (D 45′)

4.6 SNAP

Material in this section is based on or adapted from (Grenon, 2003b; Grenon and Smith, 2003).

Primitive terms

The term space designates an individual, the spatial universe. It is an independent entity in the broad sense of this term.

SpatialLocation(x,y) means that the SNAP entity x is located at the spatial region y. (This is exact location.)

InheresIn(x,y) means that the trope x inheres in the substantial y. It is direct inherence. (Defining inherence brings about too much sophistications at this stage, in addition, some features of tropes of more specific kinds which are not yet clarified may conflict with the prospective definition.)

Defined terms

SpatialRegion(x) means that x is a spatial region, i.e., a part of space.

SpatialSubsumption(x,y) means that x spatially subsumes y, i.e., the spatial location of x is a part of the spatial location of y.

Substantial(x) means that x is a substantial entity, i.e., an independent SPAN entities which does not overlap with space (substantial entities are located in space).

Substance(x) means that x is a substance, i.e., it is a maximally strongly connected substantial entity. It has a bona fide boundary.

Occupies(x,y) means that x occupies y, i.e., i) x and y (which are both substantial entities) do not overlap and neither do their respective locations, but ii) the location of x is an internal part of the location of the sum of the x and y.

Site(x) means that x is a site, i.e., it is a substantial entity *occupied by* a substance.

Trope(x) means that x is a trope, i.e., it is a SNAP entity which specifically depends on at least one substantial entity, in addition, it does not overlaps with any spatial region (but it is located in space). A number of species of the category of tropes are mentioned, though not more formally characterized than taxonomically. Their theories are still work in progress – so I don’t know what these are -- and prospectively requires a modal apparatus (unknown too).

MTrope(x) means that x is a monadic trop, i.e., it is specifically dependent on at most on one substantial entity.

RTrope(x) means that x is a relational trope, i.e., it is specifically dependent on at least two substantial entities.

Main subcategories of SNAP entities

$\text{SpatialRegion}(x) \equiv_{def} \text{Part}(x, \text{space})$ (D 46)

$\text{Substantial}(x) \equiv_{def} (\text{SnapEntity}(x) \wedge \sim\exists y\, \text{SD}(x,y) \wedge \sim\text{SpatialRegion}(x))$ (D 47)

$\text{Substance}(x) \equiv_{def} \text{Substantial}(x) \wedge \text{StronglyConnected}(x)$
$\wedge \forall y\, ((\text{Part}(x,y) \wedge \text{StronglyConnected}(y)) \rightarrow x=y)$ (D 48)

$\text{Occupies}(x,y) \equiv_{def} (\text{Substantial}(x) \wedge \text{Substantial}(y) \wedge \sim\text{Overlaps}(x,y)$
$\wedge \forall v \forall w \forall z\, ((\text{SpatialLocation}(x,v) \wedge \text{SpatialLocation}(y,w)$
$\wedge \text{Sum}(z,v,w)) \rightarrow (\sim\text{Overlaps}(v,w) \wedge \text{InternalPart}(v,z)))$ (D 49)

$\text{Site}(x) \equiv_{def} \exists y\, (\text{Substance}(y) \wedge \text{Occupies}(y,x))$ (D 50)

$\text{Trope}(x) \equiv_{def} \exists y\, \text{InheresIn}(x,y)$ (D 51)

$\text{MTrope}(x) \equiv_{def} \forall y \forall z\, ((\text{InheresIn}(x,y) \wedge \text{InheresIn}(x,z)) \rightarrow x = z)$ (D 52)

$\text{RTrope}(x) \equiv_{def} \exists y \exists z\, (\text{InheresIn}(x,y) \wedge \text{InheresIn}(x,z) \wedge \sim(y = z))$ (D 53)

$\text{SpatialSubsumption}(x,y) \equiv_{def} \forall v \forall w\, ((\text{SpatialLocation}(v,x)$
$\wedge \text{SpatialLocation}(w,z)) \rightarrow \text{Part}(w,v))$ (D 54)

Spatial location

$\forall x \forall y\, (\text{SpatialLocation}(x,y) \rightarrow (\text{SnapEntity}(x) \wedge \text{SpatialRegion}(y)))$ (A 55)

$\forall x \forall y \forall z\, ((\text{SpatialLocation}(x,y) \wedge \text{SpatialLocation}(x,z)) \rightarrow x = z)$ (A 56)

$\forall x\, (\text{SnapEntity}(x) \rightarrow \exists y\, \text{SpatialLocation}(x,y))$ (A 57)

Inherence

$\forall x \forall y\, (\text{InheresIn}(x,y) \rightarrow (\text{SnapEntity}(x) \wedge \sim\text{SpatialRegion}(x)$
$\wedge \sim\text{Substantial}(x) \wedge \text{Substantial}(y) \wedge \text{SD}(x,y)))$ (A 58)

$\forall x \forall y\, ((\text{InheresIn}(x,y) \wedge \text{SpatialLocation}(x,v)$
$\wedge \text{SpatialLocation}(y,w)) \rightarrow v = w)$ (A 59)

There are no bare particulars.
$\forall x \forall y\, ((\text{Substantial}(x) \rightarrow \exists y\, \text{InheresIn}(y,x))$ (A 60)

Subcategories of that of tropes (some examples)

PT(Trope,MTrope,RTrope) (A 61)

$\forall x\, ((\text{Trope}(x) \leftrightarrow (\text{MTrope}(x) \vee \text{RTrope}(x))$
$\wedge \sim(\text{Mtrope}(x) \wedge \text{RTrope}(x)))$ (A 61′)

SB(Trope,Function) (A 62)

$\forall x\, (\text{Function}(x) \rightarrow \text{Trope}(x))$ (A 62′)

SB(Trope,Quality) (A 63)

$\forall x\ (\text{Quality}(x) \rightarrow \text{Trope}(x))$ (A 63′)

SB(Trope,Role) (A 64)

$\forall x\ (\text{Role}(x) \rightarrow \text{Trope}(x))$ (A 64′)

DJ(Quality,Function) (A 65)

$\forall x\ \sim(\text{Quality}(x) \wedge \text{Function}(x))$ (A 65′)

DJ(Role,Function) (A 66)

$\forall x\ \sim(\text{Role}(x) \wedge \text{Function}(x))$ (A 66′)

DJ(Role,Quality) (A 67)

$\forall x\ \sim(\text{Role}(x) \wedge \text{Quality}(x))$ (A 67′)

possible axiom

SB(RTrope,Role) (A 68)

$\forall x\ (\text{Role}(x) \rightarrow \text{RTrope}(x))$ (A 68′)

Mereological and existential axioms and corollaries

Substantial entities

$\forall x \forall y\ ((\text{Substantial}(x) \wedge \text{Substantial}(y)) \rightarrow \exists z\ \text{sum}(z,x,y))$ (A 69)

$\forall x \forall y \forall z\ ((\text{Sum}(x,y,z) \wedge \text{Substantial}(y) \wedge \text{Substantial}(z))$
$\rightarrow \text{Substantial}(x))$ (A 70)

$\forall x \forall y\ (\text{Substantial}(x) \wedge \text{Part}(y,x) \rightarrow \text{Substantial}(y))$ (A 71)

Possible corollaries

$\forall x \forall y\ ((\text{Substance}(x) \wedge \text{Substance}(y) \wedge \text{Part}(x, y)) \rightarrow x=y)$ (C 72)

$\forall x\ (\text{Substantial}(x) \leftrightarrow \exists y\ (\text{Substance}(y) \wedge \text{Overlaps}(x,y)))$ (C 73)

possible axiom

$\forall x \forall y\ ((\text{Substantial}(x) \wedge \text{Part}(x,y)) \rightarrow \text{Substantial}(y))$ (A 74)

$\forall x \forall y\ ((\text{Substantial}(x) \wedge \text{Substantial}(x) \wedge \text{SpatialSubsumption}(x,y))$
$\rightarrow \text{Part}\ (y,x))$ (A 75)

Tropes

$\forall x \forall y\ ((\text{Trope}(x) \wedge \text{Trope}(y)) \rightarrow \exists z\ \text{sum}(z,x,y))$ (A 76)

$\forall x \forall y \forall z\ ((\text{Sum}(x,y,z) \wedge \text{Trope}(y) \wedge \text{Trope}(z)) \rightarrow \text{Trope}(x))$ (A 77)

$\forall x \forall y\ ((\text{Trope}(x) \wedge \text{Part}(y,x)) \rightarrow \text{Trope}(y))$ (A 78)

possible axiom

$\forall x \forall y\ ((Trope(x) \wedge Part(x,y)) \rightarrow Trope(y))$ (A 79)

Spatial regions

SnapEntity(space) (A 80)

$\forall x\ (SpatialRegion(x) \rightarrow \sim\exists y\ SD(x,y))$ (A 81)

~Substantial(space) (C 82)

$\forall x \forall y\ ((SpatialRegion(x) \wedge SpatialRegion(y)) \rightarrow \exists z\ sum(z,x,y))$ (A 83)

$\forall x \forall y \forall z\ ((Sum(x,y,z) \wedge SpatialRegion(y) \wedge SpatialRegion(z))$
$\rightarrow SpatialRegion(x))$ (A 84)

$\forall x \forall y\ ((SpatialRegion(x) \wedge Part(y,x)) \rightarrow SpatialRegion(y))$ (A 85)

possible axiom

$\forall x \forall y\ ((SpatialRegion(x) \wedge Part(x,y)) \rightarrow SpatialRegion(y))$ (A 86)

Snap entities

$\forall x \forall y \forall z\ ((Sum(x,y,z) \wedge SnapEntity(y) \wedge SnapEntity(z))$
$\rightarrow SnapEntity(x))$ (A 87)

$\forall x \forall y\ ((SnapEntity(x) \wedge Part(x,y)) \rightarrow SnapEntity(y))$ (A 88)

PT(SnapEntity,Substantial,Trope,SpatialRegion) (A 89)

$\forall x\ (SnapEntity(x) \leftrightarrow (Substantial(x) \vee Trope(x) \vee SpatialRegion(x))$
$\wedge \sim(Substantial(x) \wedge Trope(x)) \wedge \sim(Substantial(x) \wedge SpatialRegion(x))$
$\wedge \sim(SpatialRegion(x) \wedge Trope(x))))$ (A 89')

4.7 SPAN

Material in this section is based on or adapted from (Grenon, 2003b; Grenon and Smith, 2003).

Primitive terms

The term time designates an individual: the whole of time.

TemporalLocation(x,y) means that x is the temporal region at which y is (uniquely) located. (It is exact temporal location.)

The term spacetime designates an individual: the whole of spacetime.

SpatiotemporalLocation(x,y) means that x is the temporal region at which y is (uniquely) located. (It is exact spatiotemporal location.)

Before(x,y) means that the temporal instant x is earlier than the temporal instant y. (This is the minimum we need in this presentation, BFO ought to be given an interval calculus a la Allen on extended regions. In addition, generalized temporal order on non regions will be definable straightforwardly from order relations on regions and locational relations.)

Defined terms

TemporalRegion(x) means that x is a region of time, i.e., a part of time which may be extended or instantaneous (a time instant), connected to various degrees or scattered.

Time Instant(x) means that x is an instant of time, i.e., a maximally strongly connected boundary of a temporal region.

AtTime(x,y) means that x is temporally located at y and that y is an instant of time.

TemporalCollocation(x,y) means that x and y are located at the same region of time.

TemporalSubsumption(x,y) means that x temporally subsumes y, i.e., the temporal location of x is a part of the temporal location of y.

TemporalPart(x,y) means that x is a temporal part of y, i.e., x is a part of y such that all parts of y temporally collocated with x are parts of x. (It is trivial to introduce a ternary relation indicating the time of location of x)

TemporalSlice(x,y) means that x is a temporal slice of y, i.e., x is an instantaneous temporal part of y.

SpatiotemporalRegion(x) means that x is a region of spacetime, i.e., a part of spacetime.

SpatiotemporalCollocation(x,y) means that x and y are located at the same region of spacetime.

SpatiotemporalSubsumption(x,y) means that x temporally subsumes y, i.e., the spatiotemporal location of x is a part of the spatiotemporal location of y.

SpatiotemporalPart(x,y) means that x is a spatiotemporal part of y, i.e., x is a part of y such that all parts of y spatiotemporally collocated with x are parts of x.

Occurs At: OCC(x,y) means that x -- x is neither a temporal nor spatiotemporal region -- has a temporal slice located at the instant t.

Processual(x) means that x is a processual, i.e., an happening, an occurrent (not a temporal or spatiotemporal region).

Process(x) means that x is a process, i.e., a maximally strongly connected occurrent (processual).

Event(x) means that x is an event, i.e., a temporal slice of a processual.

BonaFideEvent(x) manes that x is a bona fide event, i.e., a maximally strongly connected boundary of an occurrent.

Temporal and spatiotemporal regions

SpanEntity(time) (A 90)

SpanEntity(spacetime) (A 91)

~ Overlaps(spacetime,time) (A 92)

TemporalRegion(x) $\equiv_{def}$ Part(x,time) (D 93)

TemporalInstant(x) $\equiv_{def}$ $\exists$y (TemporalRegion(y)

$\wedge$ BoundaryFor(x,y) $\wedge$ StronglyConnected(x)
$\wedge$ $\forall$z ((BoundaryFor(z,y) $\wedge$ StronglyConnected(z)) $\rightarrow$ x=z)) (D 94)

SpatiotemporalRegion(x) $\equiv_{def}$ Part(x,spacetime) (D 95)

Temporal location

$\forall$x$\forall$y (TemporalLocation(x,y)
$\rightarrow$ (SpanEntity(x)) $\wedge$ TemporalRegion(y))) (A 96)

$\forall$x$\forall$y ((TemporalLocation(x,y) $\wedge$ TemporalLocation(x,z)) $\rightarrow$ y=z) (A 97)

$\forall$x (SpanEntity(x) $\rightarrow$ $\exists$y TemporalLocation(y,x)) (A 98)

$\forall$x (TemporalRegion(x) $\rightarrow$ TemporalLocation(x,x)) (A 99)

Further definitions

AtTime(x,y) $\equiv_{def}$ (TemporalLocation(x, y) $\wedge$ TimeInstant(y)) (D 100)

TemporalColocation(x,y) $\equiv_{def}$ $\exists$z (TemporalLocation(x,z)
$\wedge$ TemporalLocation(y,x)) (D 101)

TemporalSubsumption(x,y) $\equiv_{def}$ $\forall$v$\forall$w ((TemporalLocation(x,v) $\wedge$
TemporalLocation(y,w)) $\rightarrow$ Part(w,v)) (D 102)

Spatiotemporal location

$\forall$x$\forall$y (SpatiotemporalLocation(x,y)
$\rightarrow$ (SpanEntity(x) $\wedge$ SpatiotemporalRegion(y))) (A 103)

$\forall$x$\forall$y$\forall$z ((SpatiotemporalLocation(x,y)
$\wedge$ SpatiotemporalLocation(x,z)) $\rightarrow$ y=z) (A 104)

$\forall$x (SpanEntity(x) $\rightarrow$ $\exists$y SpatiotemporalLocation(x,y)) (A 105)

$\forall$x (SpatiotemporalRegion(x) $\rightarrow$ SpatiotemporalLocation(x,x)) (A 106)

Spatiotemporal and temporal mereology

TemporalPart(x,y) $\equiv_{def}$ Part(x,y) $\wedge$ SpanEntity(x)
$\wedge$ SpanEntity(y) $\wedge$ $\forall$z ((Part(z,y) $\wedge$ TemporallyColocated(x,z))
$\rightarrow$ Part(z,x)) (D 107)

TemporalSlice(x,y) $\equiv_{def}$ TemporalPart(x,y) $\wedge$ $\exists$z AtTime(x,z) (D 108)

SpatiotemporalPart(x,y) $\equiv_{def}$ Part(x,y) $\wedge$ SpanEntity(x) $\wedge$ SpanEntity(y)
$\wedge$ $\forall$z ((Part(z,y) $\wedge$ SpatiotemporallyColocated(x,z)) $\rightarrow$ Part(z,x)) (D 109)

$\forall$x$\forall$y (TemporalPart(x,y) $\rightarrow$ TemporalSubsumption(y,x)) (C 110)

Occurrence and processuals

OccursAt(x,y) $\equiv_{def}$ (SpanEntity(x) $\wedge$ ~TemporalRegion(x)
$\wedge$~SpatiotemporalRegion(x) $\wedge$$\exists$z (TemporalSlice(z,x) $\wedge$ AtTime(z,y)) (D 111)

Processual(x) $\equiv_{def}$ $\exists$y OccursAt(x,y) (D 112)

$\text{Process}(x) \equiv_{def} \text{Processual}(x) \wedge \text{StronglyConnected}(x)$
$\wedge\ \forall y\ ((\text{Part}(x,y) \wedge \text{StronglyConnected}(y)) \rightarrow x=y)$ (D 113)

$\text{Event}(x) \equiv_{def} \exists y\ (\text{Processual}(y) \wedge \text{TemporalSlice}(x, y))$ (D 114)

$\text{Event-BF}(x) \equiv_{def} \exists y\ (\text{Processual}(y) \wedge \text{TemporalSlice}(x, y)$
$\wedge\ \text{BoundaryFor}(x, y))$ (D 115)

Temporal order (on instants)

$\forall x \forall y\ (\text{Before}(x,y) \rightarrow (\text{TemporalInstant}(x) \wedge \text{TemporalInstant}(y)))$ (A 116)

$\forall x\ {\sim}\text{Before}(x,x)$ (A 117)

$\forall x \forall y\ (\text{Before}(x,y) \rightarrow {\sim}\text{Before}(y,x))$ (A 118)

$\forall x \forall y \forall z\ ((\text{Before}(x,y) \wedge \text{Before}(y,z)) \rightarrow \text{Before}(x,z))$ (A 119)

$\forall x \forall y\ ((\text{TemporalInstant}(x) \wedge \text{TemporalInstant}(y))$
$\rightarrow (\text{Before}(x,y) \vee (x = y) \vee \text{Before}(y,x)))$ (A 120)

Mereological and existential axiom

Temporal region

$\forall x \forall y\ ((\text{TemporalRegion}(x) \wedge \text{TemporalRegion}(y))$
$\rightarrow \exists z\ \text{sum}(x,y,z))$ (A 121)

$\forall x \forall y \forall z\ ((\text{Sum}(x,y,z) \wedge \text{TemporalRegion}(y) \wedge \text{TemporalRegion}(z))$
$\rightarrow \text{TemporalRegion}(x))$ (A 122)

$\forall x \forall y\ ((\text{TemporalRegion}(x) \wedge \text{Part}(y,x)) \rightarrow \text{TemporalRegion}(y))$ (A 123)

possible axiom

$\forall x \forall y\ ((\text{TemporalRegion}(x) \wedge \text{Part}(x,y)) \rightarrow \text{TemporalRegion}(y))$ (A 124)

Spatiotemporal region

$\forall x \forall y\ ((\text{SpatiotemporalRegion}(x)$
$\wedge\ \text{SpatiotemporalRegion}(y)) \rightarrow \exists z\ \text{Sum}(x,y,z))$ (A 125)

$\forall x \forall y \forall z\ ((\text{Sum}(x,y,z) \wedge \text{SpatiotemporallRegion}(y)$
$\wedge\ \text{SpatiotemporallRegion}(z)) \rightarrow \text{SpatiotemporallRegion}(x))$ (A 126)

$\forall x \forall y\ ((\text{SpatiotemporalRegion}(x) \wedge \text{Part}(y,x))$
$\rightarrow \text{SpatiotemporalRegion}(y))$ (A 127)

$\forall x \forall y\ ((\text{SpatiotemporalRegion}(x) \wedge \text{Part}(x,y))$
$\rightarrow \text{SpatiotemporalRegion}(y))$ (A 128)

Processual entities

$\forall x \forall y\ ((\text{Processual}(x) \wedge \text{Part}(y,x)) \rightarrow \text{Processual}(y))$ (A 129)

$\forall x \forall y$ ((Processual(x) $\wedge$ Processual(y)) $\rightarrow$ $\exists z$ Sum(x,y,z)) (A 130)

$\forall x \forall y$ (Sum(x,y,z) $\wedge$ Processual(y) $\wedge$ Processual(z))
$\rightarrow$ Processual(x)) (A 131)

$\forall x \forall y$ (Process(x) $\wedge$ Process(y) $\wedge$ p(x, y)) $\rightarrow$ x=y) (A 132)

possible axiom

$\forall x \forall y$ ((Processual(x) $\wedge$ Part(x,y)) $\rightarrow$ Processual(y)) (A 133)

Span entity

PT(SpanEntity,Processual,TemporalRegion,SpatiotemporalRegion) (A 134)
$\forall x$ (SpanEntity(x) $\leftrightarrow$ (Processual(x) $\vee$ TemporalRegion(x) $\vee$
SpatiotemporalRegion(x)) $\wedge$ ~(Processual(x) $\wedge$ TemporalRegion(x)) $\wedge$
~(Processual(x) $\wedge$ SpatiotemporalRegion(x)) $\wedge$ ~(TemporalRegion(x) $\wedge$
SpatiotemporalRegion (x)))) (A 134')

$\forall x \forall y$ ((SpanEntity(x) $\wedge$ Part(y,x)) $\rightarrow$ SpanEntity(y)) (A 135)

$\forall x \forall y$ ((SpanEntity(x) $\wedge$ Part(x,y) $\rightarrow$ SpanEntity(y)) (A 136)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ SpanEntity(y) $\wedge$ SpanEntity(z))
$\rightarrow$ SpanEntity(x)) (A 137)

$\forall x$ (SpanEntity(x) $\rightarrow$ $\exists y$ TemporalPart(y,x)) (A 138)

$\forall x$ (SpanEntity(x) $\rightarrow$ $\exists y$ SpatiotemporalPart(y,x)) (A 139)

$\forall x \forall y$ ((TemporalRegion(x) $\wedge$ SpatiotemporalPart(y,x)) $\rightarrow$ x=y) (A 140)

4.8 Temporalizing BFO

Material in this section is adapted from (Grenon, 2003b; Grenon and Smith, 2003), temporalization relies on section 4.1, 4.2, 4.3, and 4.6.

In this section, certain terms, e.g., Substantial, SpatialRegion, and Trope, are taken as pseudo-primitives (entities don't jump from one of these categories to another over time). I limit the glossary to new terms.

Existence and explicitly temporalized SNAP vocabulary

Primitive terms

ExistsAt(x,y) means that the SNAP entity x exists at the time instant y.

SpatialLocationAt(x,y,z) means that at the instant of time y, x is spatially located at z.

InheresInAt(x,y,z) means that at the instant of time z, x inheres in y.

Defined terms

ExistsDuring(x,y) means that the SNAP entity x exists during the whole temporal region y. (It is a very weak claim.)

Existence

$\forall x \forall y$ (ExistsAt(x,y) $\rightarrow$ (SnapEntity(x) $\wedge$ TemporalInstant(y))) (A 141)

ExistsDuring(x,y) $\equiv_{def}$ TemporalRegion(y)
$\wedge$ $\forall z$ ((TemporalInstant(z) $\wedge$ Part(z,y)) $\rightarrow$ ExistsAt(x,z)) (D 142)

ExistsDuring(space,time) (A 143)

possible axiom

$\forall x \forall y$ (ExistsDuring(x,y) $\rightarrow$ StronglyConnected(y)) (A 144)

Temporalized spatial location.

$\forall x \forall y \forall z$ (SpatialLocationAt(x,y,z) $\rightarrow$ (SnapEntity(x)
$\wedge$ SpatialRegion(y) $\wedge$ TemporalRegion(z))) (A 145)

$\forall x \forall y \forall z$ (SpatialLocationAt(x,y,z) $\rightarrow$ (ExistsDuring(x,z)
$\wedge$ ExistsDuring(y,z))) (A 146)

$\forall x \forall y \forall z \forall w$ ((SpatialLocationAt(x,y,z) $\wedge$ SpatialLocationAt(x,w,z))
$\rightarrow$ y=w) (A 147)

Temporalized Inherence.

$\forall x \forall y \forall z$ (InheresIn(x,y,z)
$\rightarrow$ (Trope(y) $\wedge$ Substantial(y) $\wedge$ TemporalRegion(z))) (A 148)

$\forall x \forall y \forall z$ (InheresIn(x,y,z) $\rightarrow$ (ExistsDuring(x,z)
$\wedge$ ExistsDuring(y,z))) (A 149)

$\forall x \forall y \forall z$ (InheresIn(x,y,z) $\rightarrow$ SD(x,y) (A 150)

$\forall x \forall y \forall z$ ((InheresIn(x,y,z) $\wedge$ SpatialLocationAt(x,v,z)
$\wedge$ SpatialLocationAt(y,w,z))$\rightarrow$ v = w) (A 151)

$\forall x \forall y$ ((Trope(x) $\wedge$ ExistsAt(x,y)) $\leftrightarrow$ $\exists z$ InheresIn(x,z,y)) (A 152)

$\forall x \forall y$ ((Substance(x) $\wedge$ ExistsAt(x,y)) $\rightarrow$ $\exists z$ InheresIn(z,x,y)) (A 153)

$\forall x \forall y \forall v \forall w$ ((MTrope(x) $\wedge$ ExistsAt(x,y)) $\leftrightarrow$
((InheresIn(x,v,y) $\wedge$ InheresIn(x,w,y)) $\rightarrow$ v = w)) (D 154)

$\forall x \forall y$ ((RTrope(x) $\wedge$ ExistsAt(x,y)) $\leftrightarrow$ $\exists v \exists w$
(InheresIn(x,v,y) $\wedge$ InheresIn(x,w,y) $\wedge$ ~(v=w))) (D 155)

Principles of non migration for monadic and (binary) relational tropes

$\forall x \forall y \forall z \forall v \forall w$ ((MTrope(x) $\wedge$ InheresIn(x,v,y) $\wedge$ InheresIn(x,w,z)) $\rightarrow$ v=w) (A 156)

$\forall x$ (((MTrope(x) $\wedge$ $\forall y \forall z \forall v \forall w$ ((InheresIn(x,y,z) $\wedge$ InheresIn(x,v,z) $\wedge$ InheresIn(x,w,z)) $\rightarrow$ ((y=v) $\vee$ (y=w) $\vee$ (v=w)))) $\rightarrow$ $\forall y \forall z \forall u \forall v \forall z' \forall w$ ((InheresIn(x,y,z) $\wedge$ InheresIn(x,u,z) $\wedge$ InheresIn(x,v,z') $\wedge$ InheresIn(x,w,z') $\wedge$ ~(y=u) $\wedge$ ~(v=w)) $\rightarrow$ (((y=v) $\wedge$ (u=w)) $\vee$ ((y=w) $\wedge$ (u=v))))) (A 157)

Temporalizing the remaining vocabulary

Primitive terms

PartAt(x,y,z) means that x is a part of y at z. Part is the only primitive temporalized mereological relation. In the case of perdurants, z will be the time of location of x (so, it is definable in this context).

BoundaryForAt(x,y,z) means that, at z, x is a bona fide boundary for y.

FiatBoundaryForAt(x,y) means that x is a fiat boundary for y.

SDAt(x,y,z) means that x is specifically dependent on y at z.

Defined terms

As in the mereological case, the temporalized SPAN vocabulary is definable in terms of temporal or spatial location and the non temporalized primitive. All definitions based on the non temporalized primitive are given more or less straightforwardly in the expected way.

Consistently with the use of SD until now, we may define this relation as follow:

SD(x,y) $\equiv_{def}$ $\forall z$ ((ExistsAt(x,z) $\vee$ OccursAt(x,z)) $\rightarrow$ SDAt(x,y,z)) (D 158)

4.9 SNAP and SPAN

Material in this section is partially based on or adapted from (Grenon, 2003c; Grenon, 2003b; Grenon and Smith, 2003).

Primitive terms

The native primitive terms of BFO are non temporalized predicates for the relation of participation and realization which obtains at a given moment of time. ParticipatesIn(x,y) means that x participates in y (x exists at the time of obtainment of the relation and y is located at that time). The most fundamental form of participation is thus between a SNAP entity and a temporal slice of a process (an event) – (Grenon, 2003c). It is this variant but with the mention of the time of location of the event in question which I will take as primitive. The other native primitive is Realization(x,y) where x is a trope – with analogous qualifications.

ParticipatesAt(x,y,z) means that x is a substantial which participates in event y at z.

RealizedAt(x,y,z) means that x is in a process of realization in the event y at z.

Defined terms

STParticipantAt(x,y,z) means that there is an instantaneous spatiotemporal part of y at z in which x participates.

TParticipantAt(x,y,z) means that there is a temporal slice of y at z in which x participates.

CParticipantIn(x,y) means that x is a complete participant in y, i.e., x participates in each temporal slice of y at the time at which it occurs.

Life(x,y) means that x is the life of y. The life of a substantial is the fusion of all processuals it is a complete participant of.

SpatialLocalizationAt(x,y,z) means that the event x is spatially localized at the spatial region y at the instant of time z; y is the spatial location of the fusion of the location of the participants in x at z.

Functioning(x) means that x is a functioning process, i.e., the realization of a function.

Axioms for ParticipatesAt

$\forall x \forall y \forall z$ (ParticipatesAt(x,y,z)
$\rightarrow$ (ExistsAt(x,z) $\wedge$ AtTime(y,z) $\wedge$ SDAt(y,x,z))) (A 159)

$\forall x \forall y \forall z$ (ParticipatesAt(x,y,z) $\rightarrow$
(Substantial(x) $\wedge$ Event(y) $\wedge$ TemporalInstant(y))) (C 160)

possible axioms

$\forall x$ (Processual(x) $\rightarrow \exists y \exists z$ ParticipatesAt(y,x,z)) (A 161)

$\forall x$ (Substantial(x) $\rightarrow \exists y \exists z$ ParticipatesAt(x,y,z)) (A 162)

Definitions

STParticipantAt(x,y,z) $\equiv_{def}$ $\exists w$ (SpatiotemporalPart(w,y)
$\wedge$ ParticipatesAt(x,w,z)) (A 163)

TParticipantAt(x,y,z) $\equiv_{def}$ $\exists w$ (TemporalPart(w,y)
$\wedge$ ParticipatesAt(x,w,z)) (A 164)

CParticipantIn(x,y) $\equiv_{def}$ $\forall z$ (OccursAt(y,z) $\rightarrow$ TParticipantAt(x,y,z)) (A 165)

Life(x,y) $\equiv_{def}$ Fusion(x,w[CParticipantIn(y,w)]) (A 166)

SpatialLocalizationAt(x,y,z) $\equiv_{def}$ Fusion(y,w[$\exists v$ (ParticipatesAt(v,x,z)
$\wedge$ SpatialLocationAt(v,w,z))]) (A 167)

possible corollaries

$\forall x$ (Substantial(x) $\rightarrow \exists y$ Life(y,x)) (A 168)

$\forall x \forall y$ (Life(x,y) $\rightarrow$ SD(x,y)) (A 169)

possible axioms (can this at any rate characterize a kind of substantial entities?)

$\forall x \forall y \forall z \forall w$ (ParticipatesAt(x,y,z) $\wedge$ (Part(w,x))
$\rightarrow$ ParticipatesAt(w,y,z)) (A 170)

Axioms for RealizedAt

$\forall x \forall y \forall z$ (RealizedAt(x,y,z)
$\rightarrow \exists w$ (InheresIn(x,w,z) $\wedge$ ParticipatesAt(w,y,z)) (A 171)

$\forall x \forall y \forall z$ ((InheresIn(x,y,z) $\wedge$ RealizedAt(x,w,z))
$\rightarrow$ ParticipatesAt(y,w,z))) (A 172)

$\forall x \forall y \forall z \forall v \forall w$ (RealizedAt(x,y,z) $\wedge$ (InheresIn(x,v,z) $\wedge$ Life(w,z))
$\rightarrow$ Part(x,w,z)) (C 173)

Functioning(x) $\equiv_{def}$ $\exists y \exists z$ (RealizedAt(x,y,z) $\wedge$ Function(y)) (A 174)

References

References for section 4

Casati, R. and Varzi, A. C. (1996). The Structure of Spatial Localization, *Philosophical Studies*, 82: 2, 205-239.

Casati, R. and Varzi, A. C. (1999). *Parts and Places: The Structures of Spatial Representation*. Cambridge, MA: The M.I.T. Press.

Grenon (2003b) Spatio-temporality in Basic Formal Ontology: SNAP and SPAN, Upper-Level Ontology, and Framework of Formalization (Part I), *IFOMIS Technical Report Series*, 05/2003.

Grenon, P. (2003c) Spatio-temporality in BFO: Trans-ontology (Part II), manuscript.

Grenon and Smith (2003) SNAP and SPAN: Towards Dynamic Geospatial Ontology, forthcoming in *Spatial Cognition and Computation*, 4:1 (2004).

Simons, P. (1987). *Parts: A Study in Ontology*. Oxford: Clarendon Press.

Smith, B. (1997). On Substances, Accidents and Universals: In Defence of a Constituent Ontology. *Philosophical Papers* 26, 105–127.

Smith, B. (1998) The Basic Tools of Formal Ontology, in Nicola Guarino (ed.), *Formal Ontology in Information Systems* Amsterdam, Oxford, Tokyo, Washington, DC: IOS Press (Frontiers in Artificial Intelligence and Applications), 19–28.

Smith, B. and Varzi, A. C. (2000). Fiat and Bona Fide Boundaries. *Philosophy and Phenomenological Research*, 60: 2, 401–420.

2 **Formal Characterization of BFO, revised 23 12 03**

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4 Formal characterization of BFO

The language used here is a first-order one with equality. It contains the usual logical symbols (infix notation): ~ for negation, ∧ for conjunction, ∨ for disjunction, → for material implication, ↔ for logical equivalence, = for equality, ∀ (respectively ∃) for the universal (respectively existential) quantifier. I use the following conventions:

– variables (individuals): x, y, z, v, w, …

– constants (individuals): lower case strings of Latin letters

– predicates: concatenations of capitalized strings of Latin letters (prefix notation)

– In a couple of occurrences I will use schematic letters.

– I indicate tentative uses of the symbols 'PT', 'SB', and 'DJ' from (Masolo *et al.*, 2003) for partition, subsumption, and disjointness among categories (with the expected interpretations given the earlier discussions on universals). However, I also provide alternative formalization simulating subsumption and related notions via material implication (this comes very close to BFO's approach in terms of universals, one only needs to replace monadic predication with the corresponding statement using instantiation).

I do not offer a full logic, in particular there will be no consideration on a deductive system. I simply assume all the tautologies of classical predicate calculus and standard rules of inference, the alleged corollaries or theorems are so in natural deduction. All formulas are given a number by order of appearance. The number should not be given any specific signification (although, of course, new formulas are asserted in contexts which generally assume previously asserted formulas or introduction of the relevant primitives). Furthermore, 'A' next to a number indicates that the formula is an axiom, 'D' that it is a definition, 'C' that it is a putative corollary.

There is a disclaimer on the axiomatization provided here. The sources – when there is any – are somewhat heterogeneous, even in a single section (mereotopology in particular). Starting with section 4.5, I also had to draw on programmatic and sometimes merely suggested material or make new suggestions. Obviously, there must be flows and this material should be regarded as a transitory result of a work in progress.

The axiomatization is presented modularly in the spirit of BFO. In particular, SNAP is first axiomatized in a self-standing manner. Later on, a temporalized variant of this vocabulary is introduced. This allows to simulate only some of the features of the meta-ontological variant of BFO's formalism.

4.1 Mereology

Material in this section is based on or adapted from (Simons, 1987; Smith, 1997; Smith and Varzi, 2000)

Primitive term

Part(x,y) means that x is a part of y. Part is the only primitive mereological relation.

Defined terms

ProperPart(x,y) means that x is a proper part of y, i.e., x and y are distinct but x is a part of y (or a is part of y but b is not part of x).

Overlaps(x,y) means that x overlaps with y, i.e., x and y have a part in common.

Underlaps(x,y) means that x and y are both parts of a third entity.

Fusion(y,[Φx]) means that y is the fusion of the Φers, where Φ is a formula of the language with at least one free variable. I do not use an operator for definite descriptions. Rather, I systematically use relations. The definition and further uses are probably best read as axiom schemas. Uniqueness of the fusion is demanded in the definition. Existential axioms need be supplied contextually.

Sum(x,y,z) means that x is the sum of y and z, i.e., x is the fusion of all entities overlapping y or x.

Difference(x,y,z) means that x is the difference between y and z, i.e., the fusion of all parts of y which do not overlap z.

Product(x,y,z) means that x is the product of z, i.e., the fusion of all common parts of y and z.

Complement(x,y) holds when x is the complement of y, i.e., the sum of all entities which do not overlap with y. In practice this will be useless unless relative to a given category. An amended concept could be introduced by requiring the fused entities to instantiate a given universal, for instance, a cumulative one instantiated by y.

Cumulative universals (predicates) are such that the sum of their instances falls under them as well.

Dissective ones are such the parts of their instances is one of their instances as well.

Axioms for Part

$\forall x\ Part(x,x)$ (A 175)

$\forall x \forall y \forall z\ ((Part(x,y) \wedge Part(y,z)) \rightarrow Part(x,z))$ (A 176)

$\forall x \forall y\ ((Part(x,y) \wedge Part(y,x)) \rightarrow x = y)$ (A 177)

Definitions

$ProperPart(x,y) \equiv_{def} Part(x,y) \wedge \sim(x = y))$ (D 178)

$Overlaps(x,y) \equiv_{def} \exists z\ (Part(z,x) \wedge Part(z,y))$ (D 179)

$\text{Underlaps}(x,y) \equiv_{def} \exists z\ (\text{Part}(x,z) \wedge \text{Part}(y,z))$ (D 180)

$\text{Fusion}(y,x[\Phi x]) \equiv_{def} \forall z\ ((\text{Overlaps}(z,y) \leftrightarrow \exists x\ (\Phi x \wedge \text{Overlaps}(z,w)))$

$\wedge\ \forall z'\ (\text{Overlaps}(z,y) \leftrightarrow \exists x\ (\Phi x \wedge \text{Overlaps}(z,w))) \rightarrow z = z'))$ (D 181)

$\text{Sum}(x,y,z) \equiv_{def} \text{Fusion}(x,w[\text{Part}(w,y) \vee \text{Part}(w,z)])$ (D 182)

$\text{Difference}(x,y,z) \equiv_{def} \text{Fusion}(x,z[P(z,x) \wedge \sim\text{Overlaps}(z,y)])$ (D 183)

$\text{Product}(x,y,z) \equiv_{def} \text{Fusion}(x,w[\text{Part}(w,y) \wedge \text{Part}(w,z)])$ (D 184)

$\text{Complement}(x,y) \equiv_{def} \text{Fusion}(x,z[\sim\text{Overlaps}(z,y)])$ (D 185)

Two axiom schemas
Using the schematic letter '**P**' for a given predicate symbol, here are a couple of definitions adapted from (Simons, 1987):

(D 186) **P** is dissective:
$\forall x \forall y\ ((\mathbf{P}(x) \wedge \text{Part}(y,x)) \rightarrow \mathbf{P}(y))$ (D 186)

(D 187) **P** is cumulative:
$\forall x \forall y \forall z\ ((\text{Sum}(x,y,z) \wedge \mathbf{P}(x) \wedge \mathbf{P}(y)) \rightarrow \mathbf{P}(z))$ (D 187)

4.2 Temporalized Mereology: -At Relations

All these relations hold at a given instant of time.

Assumed vocabulary

TimeInstant

Primitive term

PartAt(x,y,z) means that x is a part of y at z. PartAt is the only temporalized primitive temporalized mereological relation. In the case of perdurants, z will be the time of location of x (so, it is definable in this context). See PartDuring for a cariant which holds over an extended region of time (i.e., at every instant inside it).

Defined terms

ProperPartAt(x,y,z) means that x is a proper part of y at z, i.e., at z, x and y are distinct but x is a part of y (or a is part of y but b is not part of x).

OverlapsAt(x,y,z) means that x overlaps with y at z, i.e., at z, x and y have a part in common.

UnderlapsAt(x,y,z) means that x and y are both parts of a third entity at z.

FusionAt(y,x[Φx],z) means that y is the fusion of the Φers at z, where Φ is a formula of the language with at least one free variable. I do not use an operator for definite descriptions. Rather, I systematically use relations. The definition and further uses are probably best read as axiom schemas.

Uniqueness of the fusion is demanded in the definition. Existential axioms need be supplied contextually.

SumAt(x,y,z,w) means that x is the sum of y and z at w, i.e., x is the fusion of all entities overlapping y or x at w.

DifferenceAt(x,y,z,w) means that x is the difference between y and z at w, i.e., the fusion of all parts of y which do not overlap w.

ProductAt(x,y,z,w) means that x is the product of z at w, i.e., the fusion of all common parts of y and z at w.

ComplementAt(x,y,z) holds when x is the complement of y at z, i.e., the sum of all entities which do not overlap with y. In practice this will be useless unless relative to a given category. An amended concept could be introduced by requiring the fused entities to instantiate a given universal, for instance, a cumulative one instantiated by y.

Cumulative universals (predicates) are such that the sum of their instances falls under them as well.

Dissective ones are such the parts of their instances is one of their instances as well.

Axioms for PartAt

$\forall x \forall y \ PartAt(x,x,y)$ (A 188)

$\forall x \forall y \forall z \forall w \ ((PartAt(x,y,w) \wedge PartAt(y,z,w)) \rightarrow PartAt(x,z,w))$ (A 189)

$\forall x \forall y \forall w \ ((PartAt(x,y,w) \wedge PartAt(y,x,w)) \rightarrow x = y)$ (A 190)

$\forall x \forall y \forall z \ (PartAt(x,y,z) \rightarrow TemporalInstant(z))$ (A 191)

Note: See additional constraints in section BFO.

Definitions

$ProperPartAt(x,y,z) \equiv_{def} PartAt(x,y,z) \wedge \sim(x = y))$ (D 192)

$OverlapsAt(x,y,w) \equiv_{def} \exists z \ (PartAt(z,x,w) \wedge PartAt(z,y,w))$ (D 193)

$UnderlapsAt(x,y,w) \equiv_{def} \exists z \ (PartAt(x,z,w) \wedge PartAt(y,z,w))$ (D 194)

$FusionAt(y,x[\Phi x],u) \equiv_{def} \forall z \ ((OverlapsAt(z,y,u) \leftrightarrow \exists x \ (\Phi x \wedge OverlapsAt(z,w,u)))$
$\wedge \forall z' \ (OverlapsAt(z,y,u) \leftrightarrow \exists x \ (\Phi x \wedge OverlapsAt(z,w,u))) \rightarrow z=z'))$ (D 195)

$SumAt(x,y,z,u) \equiv_{def} FusionAt(x,w[PartAt(w,y,u) \vee PartAt(w,z,u)],u)$ (D 196)

$DifferenceAt(x,y,z,u) \equiv_{def} FusionAt(x,w[PartAt(w,y,u)$
$\wedge \sim OverlapsAt(w,z,u)],u)$ (D 197)

$ProductAt(x,y,z,u) \equiv_{def} FusionAt(x,w[PartAt(w,y) \wedge Part(w,z,u)],u)$ (D 198)

$ComplementAt(x,y,u) \equiv_{def} FusionAt(x,z[\sim OverlapsAt(z,y,u)],u)$ (D 199)

4.3 Temporalized Mereology: -During Relations

All these relations hold at during a given temporal region, which may be as a degenerate case a temporal instant.

Assumed vocabulary

TimeInstant
TemporalRegion
PartAt
ProperPartAt
OverlapsAt
UnderlapsAt
FusionAt
SumAt
DifferenceAt
ProductAt
ComplementAt

Defined terms

PartDuring(x,y,z) means that x is a part of y during z. In the case of perdurants, z will be the time of location of x (so, it is definable in this context). If x (and consequently y) are SNAP entities, this means that x is a part of y at every instant within z.

ProperPartDuring(x,y,z) means that x is a proper part of y during z, i.e., during z, x and y are distinct but x is a part of y (or a is part of y but b is not part of x).

OverlapsDuring(x,y,z) means that x overlaps with y during z, i.e., during z, x and y have a part in common.

UnderlapsDuring(x,y,z) means that x and y are both parts of a third entity during z.

FusionDuring(y,x[Φx],z) means that y is the fusion of the Φers during z, where Φ is a formula of the language with at least one free variable. I do not use an operator for definite descriptions. Rather, I systematically use relations. The definition and further uses are probably best read as axiom schemas. Uniqueness of the fusion is demanded in the definition. Existential axioms need be supplied contextually.

SumDuring(x,y,z,w) means that x is the sum of y and z during w, i.e., x is the fusion of all entities overlapping y or x during w.

DifferenceDuring(x,y,z,w) means that x is the difference between y and z during w, i.e., the fusion of all parts of y which do not overlap w.

ProductDuring(x,y,z,w) means that x is the product of z during w, i.e., the fusion of all common parts of y and z during w.

ComplementDuring(x,y,z) holds when x is the complement of y during z, i.e., the sum of all entities which do not overlap with y. In practice this will be useless unless relative to a given category. An amended concept could be introduced by requiring the fused entities to instantiate a given universal, for instance, a cumulative one instantiated by y.

PartDuring(x,y,z) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,z,w) $\wedge$ TimeInstant(z)$\rightarrow$ PartAt(x,y,w)) (D 200)

ProperPartDuring(x,y,z) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,z,w) $\wedge$ TimeInstant(z)$\rightarrow$ ProperPartAt(x,y,w)) (D 201)

OverlapsDuring(x,y,z) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,z,w) $\wedge$ TimeInstant(z)$\rightarrow$ OverlapsAt(x,y,w)) (D 202)

UnderlapsDuring(x,y,z) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,z,w) $\wedge$ TimeInstant(z)$\rightarrow$ UnderlapsAt(x,y,w)) (D 203)

FusionDuring(y,x[Φx],z) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,z,w) $\wedge$ TimeInstant(z)$\rightarrow$ FusionAt(y,x[Φx],w)) (D 204)

SumDuring(x,y,z,u) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,u,w) $\wedge$ TimeInstant(z)$\rightarrow$ SumAt(y,x[Φx],w)) (D 205)

DifferenceDuring(x,y,z,u) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,u,w) $\wedge$ TimeInstant(z)$\rightarrow$ DifferenceAt(x,y,z,u)) (D 206)

ProductDuring(x,y,z,u) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,u,w) $\wedge$ TimeInstant(z)$\rightarrow$ ProductAt(x,y,z,w)) (D 207)

ComplementDuring(x,y,u) $\equiv_{def}$ TemporalRegion(z)
$\wedge$ $\forall$w ((PartAt(w,u,w) $\wedge$ TimeInstant(z)$\rightarrow$ ComplementAt(x,y,z,w)) (D 208)

4.4 Mereotopology

Material in this section is based on or adapted from (Smith, 1998; Smith and Varzi, 2000)

Primitive terms

BoundaryFor(x,y) means that x is a bona fide boundary for y; x is not necessarily the whole boundary of y, but any part of it. (Contrast with BoundaryOf to be defined.) A bona fide boundary for an entity is to be understood as a partial external delineation of that entity. Boundaries are lower dimensional entities (e.g., a section of a sphere is a boundary for a ball; a section of a circle is one for a disk; a point for a line). Bona fide boundaries are not all parts of the entities they bound, this is the case for closed entities (it is definitional for them).

FiatBoundaryFor(x,y) means that x is a fiat boundary for y. FiatBoundaryFor is the fiat counterpart of BoundaryFor. These are parts of the entities they are fiat boundaries for. A fiat boundary is for instance the delineation between two component parts of an entity (they are typically regarded as the products of convention).

Defined terms

BoundaryOf(x,y) means that x is the complete (bona fide) boundary of y. The boundary of an entity is the fusion of all entities which are (bona fide) boundaries for this entity. The boundary of an entity is therefore a boundary for that entity.

Closure(x,y) means that x is the closure of y. The closure of an entity is the sum of this entity with its boundary.

Interior(x,y) means that x is the interior of y. The interior of an entity is the difference between this entity and its closure.

WeaklyConnected(x) means that x is weakly connected, i.e., x is such that any two entities it is the sum of are such that their closure overlap. This is (Smith and Varzi, 2000)'s Connected.

MildlyConnected(x) means that x is mildly connected, i.e., x is such that any two entities it is the sum of are such that one overlaps with the closure of the other or vice versa. This is (Smith and Varzi, 2000)'s Connected*.

StronglyConnected(x) means that x is strongly connected, i.e., its interior is mildly connected.

ConnectsWith(x,y) means that x is connected to y, i.e., x and y overlap or x overlaps with the closure of y or y overlaps with the closure of x.

ExternallyConnectsWith(x,y) means that x is connected to y but they do not overlap.

Closed(x) means that x is closed, i.e., it is its own closure. A bona fide boundary – in particular, the boundary of this entity – for closed entity is a part of this entity.

InternalPart(x,y) (resp. FiatInternalPart(x,y)) means that x is a part of y and no boundary for (resp. fiat boundary for) x overlaps with y.

Boundary(x) means that x is a boundary of an entity (at least one).

FiatInternalPart(x,y) means that x is a fiat part of y.

FiatBoundary(x) means that x is a fiat boundary of some entity.

FiatConnected(x) means that x is a fiat entity which is self-connected.

Axioms for (Fiat)BoundaryFor

$\forall x \forall y \forall z$ ((Part(x,y) $\wedge$ BoundaryFor(y,z)) $\rightarrow$ BoundaryFor(x,z)) (A 209)

$\forall x \forall y$ (FiatBoundaryFor(x,y) $\rightarrow$ Part(x,y)) (A 210)

$\forall x \forall y \forall z$ ((Part(x,y) $\wedge$ FiatBoundaryFor(y,z)) $\rightarrow$ FiatBoundaryFor(x,z)) (A 211)

Definitions

BoundaryOf(x,y) $\equiv_{def}$ Fusion(x,z[BoundaryFor(z,y)]) (D 212)

Closure(x,y) $\equiv_{def}$ $\forall z$ (BoundaryOf(z,x) $\rightarrow$ Sum(x,y,z)) (D 213)

Interior(x,y) $\equiv_{def}$ $\forall z$ (Closure(z,y) $\rightarrow$ Difference(x,y,z)) (D 214)

WeaklyConnected(x) $\equiv_{def}$ $\forall y \forall z \forall v \forall w$ ((Sum(x,y,z)
$\wedge$ Closure(v,y) $\wedge$ Closure(w,z)) $\rightarrow$ Overlaps(v,w)) (D 215)

MildlyConnected(x) $\equiv_{def}$ $\forall y \forall z \forall v \forall w$ ((Sum(x,y,z) $\wedge$ Closure(v,y) $\wedge$ Closure(w,z)) $\rightarrow$ (Overlaps(v,z) $\vee$ Overlaps(w,y)) (D 216)

StronglyConnected(x) $\equiv_{def}$ $\forall y$ (Interior(y,x) $\rightarrow$ MildyConnected(y)) (D 217)

ConnectedWith(x,y) $\equiv_{def}$ $\forall v \forall w$ ((Closure(v,x) $\wedge$ Closure(w,y)) $\rightarrow$ (Overlaps(x,y) $\vee$ Overlaps(v, y) $\vee$ Overlaps(x, w)) (D 218)

ExternallyConnectsWith(x,y) $\equiv_{def}$ ConnectsWith(x,y) $\wedge$ ~Overlaps(x, y) (D 219)

Closed(x) $\equiv_{def}$ Closure(x,x) (D 220)

InternalPart(x,y) $\equiv_{def}$ Part(x,y) $\wedge$ $\forall z$ (BoundaryFor(z,y) $\rightarrow$ ~Overlaps(x,z)) (D 221)

Boundary(x) $\equiv_{def}$ $\exists y$ BoundaryFor(x,y) (D 222)

FiatInternalPart(x,y) $\equiv_{def}$ Part(x,y) $\wedge$ $\forall z$ (FiatBoundaryFor(z,y) $\rightarrow$ ~Overlaps(x,y)) (D 223)

FiatBoundary(x) $\equiv_{def}$ $\exists y$ FiatBoundaryFor(x,y) (D 224)

Additional Axioms

$\forall x \forall y$ (Closure(x,y) $\rightarrow$ Part(y,x)) (A 225)

$\forall x \forall y$ ((Closure(x,y) $\wedge$ Closure(z,x)) $\rightarrow$ Part(z,x)) (A 226)

$\forall x \forall y \forall z \forall u \forall v \forall w \forall z'$ (Sum(x, y, z) $\wedge$ Closure(u,x) $\wedge$ Closure(v,y) $\wedge$ Closure(w,z) $\wedge$ Sum(z',v,w)) $\rightarrow$ z = z') (A 227)

$\forall x \forall y$ (BoundaryOf(x,y) $\rightarrow$ BoundaryFor(x,y)) (C 228)

$\forall x \forall y$ ((Closed(x) $\wedge$ BoundaryFor(x,y)) $\rightarrow$ Part(x,y)) (C 229)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ StronglyConnected(x)) $\rightarrow$ FiatConnectedWith(y,z)) (A 230)

$\forall x$ ((Boundary(x) $\wedge$ FiatConnected(x)) $\rightarrow$ $\exists y \exists z$ (FiatConnected(y) $\wedge$ BoundaryFor(x,y) $\wedge$ InternalPart(z,y))) (A 231)

$\forall x$ ((Boundary(x) $\wedge$ FiatConnected(x)) $\rightarrow$ $\exists y \exists z$ (FiatConnected(y) $\wedge$ FiatBoundaryFor(x,y) $\wedge$ FiatInternalPart(z,y))) (A 232)

The following are held to be theorem by (Smith and Varzi, 2000)

$\forall x \forall y \forall z$ ((BoundaryFor(x,y) $\wedge$ BoundaryFor(y,z)) $\rightarrow$ BoundaryFor(x,z)) (C 233)

$\forall x \forall y \forall z$ ((BoundaryFor(x,y) $\wedge$ Complement(z,y)) $\rightarrow$ BoundaryFor(x,z)) (C 234)

$\forall x \forall y$ ~(ExternallyConnectsWith(x,y) $\wedge$ Closed(x) $\wedge$ Closed(y)) (C 235)

$\forall x \forall y \forall z$ ((FiatBoundaryFor(x,y) $\wedge$ FiatBoundaryFor(y,z))

$\rightarrow$ FiatBoundaryFor(x,z)) (C 236)

4.5 Temporalized Mereotopology: -At Relations

Primitive terms

BoundaryForAt(x,y,z) means that, at z, x is a bona fide boundary for y; x is not necessarily the whole boundary of y, but any part of it. (Contrast with BoundaryOf to be defined.) A bona fide boundary for an entity is to be understood as a partial external delineation of that entity. Boundaries are lower dimensional entities (e.g., a section of a sphere is a boundary for a ball; a section of a circle is one for a disk; a point for a line). Bona fide boundaries are not all parts of the entities they bound, this is the case for closed entities (it is definitional for them).

FiatBoundaryForAt(x,y,z) means that, at z, x is a fiat boundary for y. FiatBoundaryFor is the fiat counterpart of BoundaryFor. These are parts of the entities they are fiat boundaries for. A fiat boundary is for instance the delineation between two component parts of an entity (they are typically regarded as the products of convention).

Defined terms

BoundaryOfAt(x,y,z) means that, at z, x is the complete (bona fide) boundary of y. The boundary of an entity is the fusion of all entities which are (bona fide) boundaries for this entity. The boundary of an entity is therefore a boundary for that entity.

Closure(x,y,z) means that, at z, x is the closure of y. The closure of an entity is the sum of this entity with its boundary.

Interior(x,y,z) means that, at z, x is the interior of y. The interior of an entity is the difference between this entity and its closure.

WeaklyConnected(x,y) means that, at y, x is weakly connected, i.e., x is such that any two entities it is the sum of are such that their closure overlap. This is (Smith and Varzi, 2000)'s Connected.

MildlyConnected(x,y) means that, at y, x is mildly connected, i.e., x is such that any two entities it is the sum of are such that one overlaps with the closure of the other or vice versa. This is (Smith and Varzi, 2000)'s Connected*.

StronglyConnected(x,y) means that, at y, x is strongly connected, i.e., its interior is mildly connected.

ConnectsWith(x,y,z) means that, at z, x is connected to y, i.e., x and y overlap or x overlaps with the closure of y or y overlaps with the closure of x.

ExternallyConnectsWith(x,y,z) means that, at z, x is connected to y but they do not overlap.

ClosedAt(x,y) means that, at y, x is closed, i.e., it is its own closure. A bona fide boundary – in particular, the boundary of this entity – for closed entity is a part of this entity.

InternalPartAt(x,y,z) (resp. FiatInternalPart(x,y)) means that, at z, x is a part of y and no boundary for (resp. fiat boundary for) x overlaps with y.

BoundaryAt(x,y) means that, at y, x is a boundary of an entity (at least one).

FiatInternalPartAt(x,y,z) means that x is a fiat part of y.

FiatBoundaryAt(x,y) means that, at y, x is a fiat boundary of some entity.

FiatConnectedAt(x,y) means that, at y, x is a fiat entity which is self-connected.

Axioms for (Fiat)BoundaryForAt

$\forall x \forall y \forall z \forall u$ ((PartAt(x,y,u) $\wedge$ BoundaryForAt(y,z,u)
$\rightarrow$ BoundaryForAt(x,z,u)) (A 237)

$\forall x \forall y \forall z$ (FiatBoundaryForAt(x,y,z) $\rightarrow$ PartAt(x,y,z)) (A 238)

$\forall x \forall y \forall z \forall w$ ((PartAt(x,y,w) $\wedge$ FiatBoundaryForAt(y,z,w))
$\rightarrow$ FiatBoundaryForAt(x,z,w)) (A 239)

Definitions

BoundaryOfAt(x,y,u) $\equiv_{def}$ FusionAt(x,z[BoundaryFor(z,y,u)],u) (D 240)

ClosureAt(x,y,u) $\equiv_{def}$ $\forall z$ (BoundaryOfAt(z,x,u) $\rightarrow$ SumAt(x,y,z,u)) (D 241)

InteriorAt(x,y,u) $\equiv_{def}$ $\forall z$ (ClosureAt(z,y,u) $\rightarrow$ DifferenceAt(x,y,z,u)) (D 242)

WeaklyConnectedAt(x,u) $\equiv_{def}$ $\forall y \forall z \forall v \forall w$ ((SumAt(x,y,z,u)
$\wedge$ ClosureAt(v,y,u) $\wedge$ ClosureAt(w,z,u)) $\rightarrow$ OverlapsAt(v,w,u)) (D 243)

MildlyConnectedAt(x,u) $\equiv_{def}$ $\forall y \forall z \forall v \forall w$ ((SumAt(x,y,z,u)
$\wedge$ ClosureAt(v,y,u) $\wedge$ ClosureAt(w,z,u)) $\rightarrow$ (OverlapsAt(v,z,u)
$\vee$ OverlapsAt(w,y,u)) (D 244)

StronglyConnected(x,u) $\equiv_{def}$ $\forall y$ (InteriorAt(y,x,u)
$\rightarrow$ MildyConnectedAt(y,u)) (D 245)

ConnectedWithAt(x,y,u) $\equiv_{def}$ $\forall v \forall w$ ((ClosureAt(v,x,u) $\wedge$ ClosureAt(w,y,u))
$\rightarrow$ (OverlapsAt(x,y,u) $\vee$ OverlapsAt(v,y,u) $\vee$ OverlapsAt(x,w,u)) (D 246)

ExternallyConnectsWithAt(x,y,u)
$\equiv_{def}$ ConnectsWithAt(x,y,u) $\wedge$ ~OverlapsAt(x,y,u) (D 247)

ClosedAt(x,u) $\equiv_{def}$ ClosureAt(x,x,u) (D 248)

InternalPartAt(x,y,u) $\equiv_{def}$ PartAt(x,y,u)
$\wedge$ $\forall z$ (BoundaryForAt(z,y,u) $\rightarrow$ ~OverlapsAt(x,z,u)) (D 249)

BoundaryAt(x,u) $\equiv_{def}$ $\exists y$ BoundaryForAt(x,y,u) (D 250)

FiatInternalPartAt(x,y,u) $\equiv_{def}$ PartAt(x,y,u)
$\wedge$ $\forall z$ (FiatBoundaryForAt(z,y,u) $\rightarrow$ ~OverlapsAt(x,y,u)) (D 251)

FiatBoundaryAt(x,u) $\equiv_{def}$ $\exists y$ FiatBoundaryForAt(x,y,u) (D 252)

Additional Axioms

$\forall x \forall y \forall u$ (ClosureAt(x,y,u) $\rightarrow$ PartAt(y,x,u)) (A 253)

$\forall x \forall y \forall u$ ((ClosureAt(x,y,u) $\wedge$ Closure(z,x,u)) $\rightarrow$ Part(z,x,u)) (A 254)

$\forall x \forall y \forall z \forall u \forall v \forall w \forall z' \forall t$ (SumAt(x,y,z,t) $\wedge$ ClosureAt(u,x,t)
$\wedge$ ClosureAt(v,y,t) $\wedge$ ClosureAt(w,z,t) $\wedge$ SumAt(z',v,w,t)) $\rightarrow$ z = z') (A 255)

$\forall x \forall y \forall u$ (BoundaryOfAt(x,y,u) $\rightarrow$ BoundaryForAt(x,y,u)) (C 256)

$\forall x \forall y \forall u$ ((ClosedAt(x,u) $\wedge$ BoundaryForAt(x,y,u)) $\rightarrow$ PartAt(x,y,u)) (C 257)

$\forall x \forall y \forall z \forall u$ ((SumAt(x,y,z,u) $\wedge$ StronglyConnectedAt(x,u))
$\rightarrow$ FiatConnectedWithAt(y,z,u)) (A 258)

$\forall x \forall u$ ((BoundaryAt(x,u) $\wedge$ FiatConnectedAt(x,u))
$\rightarrow \exists y \exists z$ (FiatConnectedAt(y,u) $\wedge$ BoundaryForAt(x,y,u)
$\wedge$ InternalPartAt(z,y,u))) (A 259)

$\forall x \forall u$ ((BoundaryAt(x,u) $\wedge$ FiatConnectedAt(x,u)) $\rightarrow \exists y \exists z$
(FiatConnectedAt(y,u) $\wedge$ FiatBoundaryForAt(x,y,u)
$\wedge$ FiatInternalPartAt(z,y,u))) (A 260)

Possible theorems in view of adaptation from (Smith and Varzi, 2000)

$\forall x \forall y \forall z \forall u$ ((BoundaryForAt(x,y,u) $\wedge$ BoundaryForAt(y,z,u))
$\rightarrow$ BoundaryForAt(x,z,u)) (C 261)

$\forall x \forall y \forall z \forall u$ ((BoundaryForAt(x,y,u) $\wedge$ ComplementAt(z,y,u))
$\rightarrow$ BoundaryForAt(x,z,u)) (C 262)

$\forall x \forall y \forall u$ ~(ExternallyConnectsWithAt(x,y,u) $\wedge$ ClosedAt(x,u)
$\wedge$ ClosedAt(y,u)) (C 263)

$\forall x \forall y \forall z \forall u$ ((FiatBoundaryForAt(x,y,u) $\wedge$ FiatBoundaryForAt(y,z,u))
$\rightarrow$ FiatBoundaryForAt(x,z,u)) (C 264)

4.6 Dependence

Material in this section is based on or adapted from (Smith, 1997; Smith, 1998)

Primitive term

SD(x,y) means that x is specifically dependent on y. Specific dependence is defined by (Smith, 1997) modally and SD(x,y) means that x and y do not overlap and x is such that it necessitates the existence of y in order to exist. Notice in particular that specific dependence is then not a form of parthood. Here, without a modal language, I am taking dependence as primitive.

Defined terms

MSD(x,y) means that x is specifically dependent on y and are distinct entity and y is specifically dependent on x.

OSD(x,y) means that x is specifically dependent on y but y is not dependent on a.

$\forall x \forall y\ (SD(x,y) \rightarrow \sim Overlaps(x,y))$ (A 265)

$MSD(x,y) \equiv_{def} SD(x,y) \wedge SD(y,x)$ (D 266)

$OSD(x,y) \equiv_{def} SD(x,y) \wedge \sim SD(y,x)$ (D 267)

Note : These are all relations among particulars.

4.7 Temporalized Dependence

Temporalized variant for dsependence relations

Primitive term

SDAt(x,y) means that x is specifically dependent on y at z. Specific dependence at a time is not a form of parthood at a time.

Defined terms

MSDAt(x,y) means that x is specifically dependent on y and are distinct entity and y is specifically dependent on x.

OSDAt(x,y) means that x is specifically dependent on y but y is not dependent on a.

$\forall x \forall y \forall u\ (SDAt(x,y,u) \rightarrow \sim OverlapsAt(x,y,u))$ (A 268)

$MSDAt(x,y,u) \equiv_{def} SDAt(x,y,u) \wedge SDAt(y,x,u)$ (D 269)

$OSDAt(x,y,u) \equiv_{def} SDAt(x,y,u) \wedge \sim SDAt(y,x,u)$ (D 270)

Possible definition for SD (assumes ExistsAt and OccursAt)

$SD(x,y) \equiv_{def} \forall z\ ((ExistsAt(x,z) \vee OccursAt(x,z)) \rightarrow SDAt(x,y,z))$ (D 271)

4.8 Location

Actually, I will give no general theory of location or regions. This is to keep in line with the modular spirit of BFO and following (Grenon, 2003b). Each of SNAP and SPAN have their theories of location and their adequate primitive. Practically, here, I will only use exact location. A general theory of location can be extrapolated from Casati and Varzi's treatment, in particular their (1996) and (1999).
In the case of the static account of SNAP, location will be a binary relation, while in the case of its temporally sensitive treatment, a ternary one (location in space at a time).

4.9 BFO

In keeping with the modular framework of BFO, following (Grenon, 2003b) I take SNAP and SPAN entities as primitive notions. We can always introduce the term 'Entity' as applying to entities of any of the kinds used here. Since this is only a partial rendition of BFO (most importantly not including universals), I leave it open whether SpanEntity and SpanEntity form partitions of this putative Entity, i.e., whether there are any other kinds of entities. Here, the instances of SnapEntity and SpanEntity are all particulars.

SnapEntity(x) means that x is a SNAP entity.

SpanEntity(x) means that x is a SPAN entity.

DJ(SnapEntity,SpanEntity) (D 272)

$\forall x \sim(\text{SnapEntity}(x) \wedge \text{SpanEntity}(x))$ (D 45′)

Some constariants on PartAt and PartDuring

This assumes ExistsAt from the temporalized SNAP section.

$\forall x \forall y \forall z\ (\text{PartAt}(x,y,z) \rightarrow ((\text{SnapEntity}(x) \vee \text{SpanEntity}(x)) \wedge (\text{SnapEntity}(y) \vee \text{SpanEntity}(y))))$ (A 273)

$\forall x \forall y \forall z \forall w\ ((\text{PartAt}(x,y,z) \wedge (\text{SnapEntity}(x) \vee \text{SnapEntity}(y)) \wedge \sim\text{TemporalInstant}(z) \wedge \text{PartAt}(w,z,w) \wedge \text{TemporalInstant}(w)) \rightarrow \text{PartAt}(x,y,w))$ (A 274)

$\forall x \forall y \forall z\ ((\text{PartAt}(x,y,z) \wedge \text{SnapEntity}(x)) \rightarrow \text{SnapEntity}(y))$ (A 275)

$\forall x \forall y \forall z\ ((\text{PartAt}(x,y,z) \wedge \text{SnapEntity}(y)) \rightarrow \text{SnapEntity}(x))$ (A 276)

$\forall x \forall y \forall z\ ((\text{PartAt}(x,y,z) \wedge \text{SpanEntity}(x)) \rightarrow \text{SpanEntity}(y))$ (A 277)

$\forall x \forall y \forall z\ ((\text{PartAt}(x,y,z) \wedge \text{SpanEntity}(y)) \rightarrow \text{SpanEntity}(x))$ (A 278)

$\forall x \forall y \forall z\ ((\text{PartAt}(x,y,z) \wedge \text{SpanEntity}(x)) \rightarrow (\text{TemporalLocation}(x,z))$ (A 279)

$\forall x \forall y \forall z\ ((\text{PartAt}(x,y,z) \wedge \text{SnapEntity}(x)) \rightarrow (\text{ExistsAt}(x,z) \wedge \text{ExistsAt}(y,z)))$ (A 280)

$\forall x \forall y \forall z\ ((\text{PartDuring}(x,y,z) \wedge \text{SnapEntity}(x)) \rightarrow (\text{ExistsDuring}(x,z) \wedge \text{ExistsDuring}(y,z)))$

4.10 Temporalized SNAP

I will only give -At variants of this vocabulary. –During variants are straightfowardly defined as holding over intervals at each instant of which holds the –At corresponding variant.

Assumed vocabulary

Temporalized mereology and mereotopology
TimeInstant

Primitive terms

The term space designates an individual, the spatial universe. It is an independent entity in the broad sense of this term.

ExistsAt(x,y) means that the SNAP entity x exists at the time instant y.

SpatialLocationAt(x,y,z) means that at the instant of time y, x is spatially (exactly) located at z. Only SNAP entities are thus located (see in contrast SpatialLocalizationAt).

InheresInAt(x,y,z) means that at the instant of time z, x (directly) inheres in y. x is a trope (quality and so on) of y which is a substantial. Inherence is a form of specific depencdence.

Defined terms

ExistsDuring(x,y) means that the SNAP entity x exists during the whole temporal region y. (It is a very weak claim.)

SpatialRegion(x) means that x is a spatial region, i.e., a part of space.

SpatialSubsumptionAt(x,y,z) means that x spatially subsumes y at z, i.e., at z, the spatial location of x is a part of the spatial location of y.

Substantial(x) means that x is a substantial entity, i.e., an independent SPAN entities which does not overlap with space (substantial entities are located in space).

SubstanceAt(x,y) means that, at y, x is a substance, i.e., it is a maximally strongly connected substantial entity. It has a bona fide boundary.

Substance(x) means that x is a substance at every time instant at which it exists.

OccupiesAt(x,y,z) means that x occupies y at z, i.e., i) x and y (which are both substantial entities) do not overlap at z and neither do their respective locations, but ii) at z, the location of x is an internal part of the location of the sum of the x and y.

SiteAt(x,y) means that x is a site at y, i.e., it is a substantial entity occupied at y by a substance.

Trope(x) means that x is a trope, i.e., it is a SNAP entity which specifically depends on at least one substantial entity, in addition, it does not overlaps with any spatial region (but it is located in space). A number of species of the category of tropes are mentioned, though not more formally characterized than taxonomically. Their theories are still work in progress – so I don't know what these are -- and prospectively requires a modal apparatus (unknown too).

MTrope(x) means that x is a monadic trop, i.e., it is specifically dependent on at most on one substantial entity.

RTrope(x) means that x is a relational trope, i.e., it is specifically dependent on at least two substantial entities.

Existence At and During

$\forall x \forall y\ (\mathrm{ExistsAt}(x,y) \rightarrow (\mathrm{SnapEntity}(x) \wedge \mathrm{TemporalInstant}(y)))$ (A 281)

$ExistsDuring(x,y) \equiv_{def} TemporalRegion(y)$
$\wedge \forall z\ ((TemporalInstant(z) \wedge Part(z,y)) \rightarrow ExistsAt(x,z))$ (D 282)

$ExistsDuring(space,time)$ (A 283)

Note: It is open whether there are additional eternal spatial regions.

possible axiom

$\forall x \forall y\ (ExistsDuring(x,y) \rightarrow StronglyConnected(y))$ (A 284)

Temporalized spatial location: -At relation

$\forall x \forall y \forall z\ (SpatialLocationAt(x,y,z) \rightarrow (SnapEntity(x)$
$\wedge SpatialRegion(y) \wedge TemporalInstant(z)))$ (A 285)

$\forall x \forall y \forall z\ (SpatialLocationAt(x,y,z) \rightarrow (ExistsAt(x,z)$
$\wedge ExistsAt(y,z)))$ (A 286)

$\forall x \forall y \forall z \forall w\ ((SpatialLocationAt(x,y,z) \wedge SpatialLocationAt(x,w,z))$
$\rightarrow y=w)$ (A 287)

$\forall x \forall y\ ((SnapEntity(x) \wedge ExistsAt(x,y)) \rightarrow \exists z\ SpatialLocation(x,z,y))$ (A 288)

$SpatialSubsumptionAt(x,y,z) \equiv_{def} \forall v \forall w\ ((SpatialLocationAt(x,w,z)$
$\wedge SpatialLocationAt(y,w,z)) \rightarrow PartAt(w,v,z))$ (D 289)

Temporalized Inherence.

$\forall x \forall y \forall z\ (InheresInAt(x,y,z) \rightarrow (SnapEntity(x) \wedge \sim SpatialRegion(x) \wedge$
$\sim Substantial(x) \wedge Substantial(y) \wedge Substantial(y) \wedge TemporalInstant(z) \wedge$
$SDAt(x,y,z)))$ (A 290)

Note: This could perhaps be a definition.

$\forall x \forall y \forall z\ (InheresInAt(x,y,z) \rightarrow (ExistsAt(x,z)$
$\wedge ExistsAt(y,z)))$ (A 291)

Note: This will be a corollary.

$\forall x \forall y \forall z\ ((Mtrope(x) \wedge InheresInAt(x,y,z) \wedge SpatialLocationAt(x,v,z)$
$\wedge SpatialLocationAt(y,w,z)) \rightarrow v=w)$ (C 292)

$\forall x \forall y \forall z \forall v \forall w\ ((Trope(x) \wedge Fusion(w,y[InheresInAt(x,y,z)],z)$
$\wedge SpatialLocationAt(w,v,z)) \rightarrow SpatialLocationAt(x,v,z))$ (A 293)

$\forall x \forall y\ ((Trope(x) \wedge ExistsAt(x,y)) \leftrightarrow \exists z\ InheresInAt(x,z,y))$ (A 294)

$\forall x \forall y\ ((Substance(x) \wedge ExistsAt(x,y)) \rightarrow \exists z\ InheresInAt(z,x,y))$ (A 295)

Principles of non migration for monadic and (binary) relational tropes

$\forall x \forall y \forall z \forall v \forall w\ ((Mtrope(x) \wedge InheresInAt(x,v,y) \wedge InheresInAt(x,w,z))$

$\rightarrow$ v=w) (A 296)

$\forall$x (((Mtrope(x) $\wedge$ $\forall$y$\forall$z$\forall$v$\forall$w ((InheresInAt(x,y,z) $\wedge$ InheresInAt(x,v,z) $\wedge$ InheresInAt(x,w,z)) $\rightarrow$ ((y=v) $\vee$ (y=w) $\vee$ (v=w)))) $\rightarrow$ $\forall$y$\forall$z$\forall$u$\forall$v$\forall$z′$\forall$w ((InheresInAt(x,y,z) $\wedge$ InheresInAt(x,u,z) $\wedge$ InheresInAt(x,v,z′) $\wedge$ InheresInAt(x,w,z′) $\wedge$ ~(y=u) $\wedge$ ~(v=w)) $\rightarrow$ (((y=v) $\wedge$ (u=w)) $\vee$ ((y=w) $\wedge$ (u=v))))) (A 297)

Main subcategories of SNAP entities

SpatialRegion(x) $\equiv_{def}$ $\forall$y (ExistsAt(x,y) $\rightarrow$ PartAt(x,space,y) (D 298)

Substantial(x) $\equiv_{def}$ (SnapEntity(x) $\wedge$ ~$\exists$y$\exists$z SDAt(x,y,z) $\wedge$ ~SpatialRegion(x)) (D 299)

SubstanceAt(x,y) $\equiv_{def}$ Substantial(x) $\wedge$ StronglyConnectedAt(x,y) $\wedge$ $\forall$z ((PartAt(x,z,y) $\wedge$ StronglyConnectedAt(z,y)) $\rightarrow$ x=z) (D 300)

Substance(x) $\equiv_{def}$ $\forall$y (ExistsAt(x,y) $\rightarrow$ SubstanceAt(x,y)) (D 301)

OccupiesAt(x,y) $\equiv_{def}$ (Substantial(x) $\wedge$ Substantial(y) $\wedge$ ~Overlaps(x,y) $\wedge$ $\forall$v$\forall$w$\forall$z ((SpatialLocation(x,v) $\wedge$ SpatialLocation(y,w) $\wedge$ Sum(z,v,w)) $\rightarrow$ (~Overlaps(v,w) $\wedge$ InternalPart(v,z))) (D 302)

SiteAt(x) $\equiv_{def}$ $\exists$y (Substance(y) $\wedge$ Occupies(y,x)) (D 303)

Trope(x) $\equiv_{def}$ $\exists$y$\exists$z InheresInAt(x,y,z) (D 304)

$\forall$x$\forall$y ((Trope(x) $\wedge$ ExistsAt(x,y)) $\rightarrow$ $\exists$z InheresInAt(x,z,y)) (A 305)

Mtrope(x) $\equiv_{def}$ Trope(x) $\wedge$ $\forall$u$\forall$y$\forall$z ((InheresInAt(x,y,u) $\wedge$ InheresIn(x,z,u))$\rightarrow$ x=z) (D 306)

$\forall$x$\forall$y$\forall$z ((MTrope(x) $\wedge$ InheresInAt(x,y,z) $\wedge$ SpatialLocationAt(x,v,z) $\wedge$ SpatialLocationAt(y,w,z))$\rightarrow$ v=w) (C 307)

RTrope(x) $\equiv_{def}$ Trope(x) $\wedge$ $\forall$u (ExistsAt(x,u) $\rightarrow$ $\exists$y$\exists$z (InheresIn(x,y) $\wedge$ InheresIn(x,z) $\wedge$ ~(y = z)) (D 308)

There are no bare particulars.
$\forall$x$\forall$y$\forall$z ((Substantial(x) $\wedge$ ExistsAt(x,z))$\rightarrow$ $\exists$y InheresInAt(y,x.z)) (A 309)

Subcategories of that of tropes (some examples)

PT(Trope,MTrope,RTrope) (A 310)

$\forall$x ((Trope(x) $\leftrightarrow$ (MTrope(x) $\vee$ RTrope(x)) $\wedge$ ~(Mtrope(x) $\wedge$ RTrope(x))) (A 61′)

SB(Trope,Function) (A 311)

$\forall$x (Function(x) $\rightarrow$ Trope(x)) (A 62′)

SB(Trope,Quality) (A 312)

$\forall$x (Quality(x) $\rightarrow$ Trope(x)) (A 63′)

SB(Trope,Role) (A 313)

$\forall x$ (Role(x) $\rightarrow$ Trope(x)) (A 64′)

DJ(Quality,Function) (A 314)

$\forall x$ ~(Quality(x) $\wedge$ Function(x)) (A 65′)

DJ(Role,Function) (A 315)

$\forall x$ ~(Role(x) $\wedge$ Function(x)) (A 66′)

DJ(Role,Quality) (A 316)

$\forall x$ ~(Role(x) $\wedge$ Quality(x)) (A 67′)

possible axiom

SB(RTrope,Role) (A 317)

$\forall x$ (Role(x) $\rightarrow$ RTrope(x)) (A 68′)

Mereological and existential axioms and corollaries

Substantial entities

$\forall x \forall y \forall u$ ((Substantial(x) $\wedge$ Substantial(y) $\wedge$ ExistsAt(x,u) $\wedge$ ExistsAt(y,u))
$\rightarrow \exists z$ SumAt(z,x,y,u)) (A 318)

$\forall x \forall y \forall z \forall u$ ((SumAt(x,y,z,u) $\wedge$ Substantial(y) $\wedge$ Substantial(z))
$\rightarrow$ Substantial(x)) (A 319)

$\forall x \forall y \forall u$ (Substantial(x) $\wedge$ PartAt(y,x,u) $\rightarrow$ Substantial(y)) (A 320)

Possible corollaries

$\forall x \forall y \forall u$ ((Substance(x) $\wedge$ Substance(y) $\wedge$ PartAt(x,y,u)) $\rightarrow$ x=y) (C 321)

$\forall x \forall u$ ((Substantial(x) $\wedge$ ExistsAt(x,u)) $\leftrightarrow \exists y$ (SubstanceAt(y,u)
$\wedge$ OverlapsAt(x,y,u))) (C 322)

possible axiom

$\forall x \forall y \forall u$ ((Substantial(x) $\wedge$ PartAt(x,y,u)) $\rightarrow$ Substantial(y)) (A 323)

$\forall x \forall y \forall u$ ((Substantial(x) $\wedge$ Substantial(x) $\wedge$ SpatialSubsumption(x,y,u))
$\rightarrow$ PartAt(y,x,u)) (A 324)

TEMPORAL QUALIFICATION ON THE SAME MODEL AS WITH

SUBSTANTIAL ENTITIES

Tropes

$\forall x \forall y$ ((Trope(x) $\wedge$ Trope(y)) $\rightarrow \exists z$ sum(z,x,y)) (A 325)

$\forall x \forall y \forall z\ ((\mathrm{Sum}(x,y,z) \wedge \mathrm{Trope}(y) \wedge \mathrm{Trope}(z)) \rightarrow \mathrm{Trope}(x))$ (A 326)

$\forall x \forall y\ ((\mathrm{Trope}(x) \wedge \mathrm{Part}(y,x)) \rightarrow \mathrm{Trope}(y))$ (A 327)

possible axiom

$\forall x \forall y\ ((\mathrm{Trope}(x) \wedge \mathrm{Part}(x,y)) \rightarrow \mathrm{Trope}(y))$ (A 328)

Spatial regions

$\mathrm{SnapEntity}(\mathrm{space})$ (A 329)

$\forall x\ (\mathrm{SpatialRegion}(x) \rightarrow \sim\exists y\ \mathrm{SD}(x,y))$ (A 330)

$\sim\mathrm{Substantial}(\mathrm{space})$ (C 331)

$\forall x \forall y\ ((\mathrm{SpatialRegion}(x) \wedge \mathrm{SpatialRegion}(y)) \rightarrow \exists z\ \mathrm{Sum}(z,x,y))$ (A 332)

$\forall x \forall y \forall z\ ((\mathrm{Sum}(x,y,z) \wedge \mathrm{SpatialRegion}(y) \wedge \mathrm{SpatialRegion}(z))$
$\rightarrow \mathrm{SpatialRegion}(x))$ (A 333)

$\forall x \forall y\ ((\mathrm{SpatialRegion}(x) \wedge \mathrm{Part}(y,x)) \rightarrow \mathrm{SpatialRegion}(y))$ (A 334)

possible axiom

$\forall x \forall y\ ((\mathrm{SpatialRegion}(x) \wedge \mathrm{Part}(x,y)) \rightarrow \mathrm{SpatialRegion}(y))$ (A 335)

Snap entities

$\forall x \forall y \forall z\ ((\mathrm{Sum}(x,y,z) \wedge \mathrm{SnapEntity}(y) \wedge \mathrm{SnapEntity}(z))$
$\rightarrow \mathrm{SnapEntity}(x))$ (A 336)

$\forall x \forall y\ ((\mathrm{SnapEntity}(x) \wedge \mathrm{Part}(x,y)) \rightarrow \mathrm{SnapEntity}(y))$ (A 337)

$\mathrm{PT}(\mathrm{SnapEntity},\mathrm{Substantial},\mathrm{Trope},\mathrm{SpatialRegion})$ (A 338)

$\forall x\ (\mathrm{SnapEntity}(x) \leftrightarrow (\mathrm{Substantial}(x) \vee \mathrm{Trope}(x) \vee \mathrm{SpatialRegion}(x))$
$\wedge \sim(\mathrm{Substantial}(x) \wedge \mathrm{Trope}(x)) \wedge \sim(\mathrm{Substantial}(x) \wedge \mathrm{SpatialRegion}(x))$
$\wedge \sim(\mathrm{SpatialRegion}(x) \wedge \mathrm{Trope}(x))))$ (A 89')

4.11 SPAN

Material in this section is based on or adapted from (Grenon, 2003b; Grenon and Smith, 2003).

Primitive terms

The term time designates an individual: the whole of time.

TemporalLocation(x,y) means that x is the temporal region at which y is (uniquely) located. (It is exact temporal location.)

The term spacetime designates an individual: the whole of spacetime.

SpatiotemporalLocation(x,y) means that x is the temporal region at which y is (uniquely) located. (It is exact spatiotemporal location.)

Before(x,y) means that the temporal instant x is earlier than the temporal instant y. (This is the minimum we need in this presentation, BFO ought to be given an interval calculus a la Allen on extended regions. In addition, generalized temporal order on non regions will be definable straightforwardly from order relations on regions and locational relations.)

Defined terms

TemporalRegion(x) means that x is a region of time, i.e., a part of time which may be extended or instantaneous (a time instant), connected to various degrees or scattered.

Time Instant(x) means that x is an instant of time, i.e., a maximally strongly connected boundary of a temporal region.

AtTime(x,y) means that x is temporally located at y and that y is an instant of time.

TemporalCollocation(x,y) means that x and y are located at the same region of time.

TemporalSubsumption(x,y) means that x temporally subsumes y, i.e., the temporal location of x is a part of the temporal location of y.

TemporalPart(x,y) means that x is a temporal part of y, i.e., x is a part of y such that all parts of y temporally collocated with x are parts of x. (It is trivial to introduce a ternary relation indicating the time of location of x)

TemporalSlice(x,y) means that x is a temporal slice of y, i.e., x is an instantaneous temporal part of y.

SpatiotemporalRegion(x) means that x is a region of spacetime, i.e., a part of spacetime.

SpatiotemporalCollocation(x,y) means that x and y are located at the same region of spacetime.

SpatiotemporalSubsumption(x,y) means that x temporally subsumes y, i.e., the spatiotemporal location of x is a part of the spatiotemporal location of y.

SpatiotemporalPart(x,y) means that x is a spatiotemporal part of y, i.e., x is a part of y such that all parts of y spatiotemporally collocated with x are parts of x.

Processual(x) means that x is a processual, i.e., an happening, an occurrent (not a temporal or spatiotemporal region).

Process(x) means that x is a process, i.e., a maximally strongly connected occurrent (processual).

Event(x) means that x is an event, i.e., a temporal slice of a processual.

BonaFideEvent(x) manes that x is a bona fide event, i.e., a maximally strongly connected boundary of an occurrent.

$\forall x \forall y \forall z$ (SpanEntity(x) $\wedge$ SpanEntity(y)
$\rightarrow$ (Part(x,y) $\leftrightarrow$ $\exists z$ PartAt(x,y,z))) (A 339)

Temporal and spatiotemporal regions

SpanEntity(time) (A 340)

SpanEntity(spacetime) (A 341)

~ Overlaps(spacetime,time) (A 342)

TemporalRegion(x) $\equiv_{def}$ Part(x,time) (D 343)

TemporalInstant(x) $\equiv_{def}$ ∃y (TemporalRegion(y)
∧ BoundaryFor(x,y) ∧ StronglyConnected(x)
∧ ∀z ((BoundaryFor(z,y) ∧ StronglyConnected(z)) → x=z)) (D 344)

SpatiotemporalRegion(x) $\equiv_{def}$ Part(x,spacetime) (D 345)

Temporal location

∀x∀y (TemporalLocation(x,y)
→ (SpanEntity(x)) ∧ TemporalRegion(y))) (A 346)

∀x∀y ((TemporalLocation(x,y) ∧ TemporalLocation(x,z)) → y=z) (A 347)

∀x (SpanEntity(x) → ∃y TemporalLocation(x,y)) (A 348)

∀x (TemporalRegion(x) → TemporalLocation(x,x)) (A 349)

Further definitions

AtTime(x,y) $\equiv_{def}$ (TemporalLocation(x,y) ∧ TimeInstant(y)) (D 350)

TemporalColocation(x,y) $\equiv_{def}$ ∃z (TemporalLocation(x,z)
∧ TemporalLocation(y,z)) (D 351)

TemporalSubsumption(x,y) $\equiv_{def}$ ∀v∀w ((TemporalLocation(x,v) ∧
TemporalLocation(y,w)) → Part(w,v)) (D 352)

Spatiotemporal location

∀x∀y (SpatiotemporalLocation(x,y)
→ (SpanEntity(x) ∧ SpatiotemporalRegion(y))) (A 353)

∀x∀y∀z ((SpatiotemporalLocation(x,y)
∧ SpatiotemporalLocation(x,z)) → y=z) (A 354)

∀x (SpanEntity(x) → ∃y SpatiotemporalLocation(x,y)) (A 355)

∀x (SpatiotemporalRegion(x) → SpatiotemporalLocation(x,x)) (A 356)

Spatiotemporal and temporal mereology

TemporalPart(x,y) $\equiv_{def}$ Part(x,y) ∧ SpanEntity(x)
∧ SpanEntity(y) ∧ ∀z ((Part(z,y) ∧ TemporallyColocated(x,z))
→ Part(z,x)) (D 357)

TemporalSlice(x,y) $\equiv_{def}$ TemporalPart(x,y) ∧ ∃z AtTime(x,z) (D 358)

SpatiotemporalPart(x,y) $\equiv_{def}$ Part(x,y) ∧ SpanEntity(x) ∧ SpanEntity(y)
∧ ∀z ((Part(z,y) ∧ SpatiotemporallyColocated(x,z)) → Part(z,x)) (D 359)

$\forall x \forall y$ (TemporalPart(x,y) $\rightarrow$ TemporalSubsumption(y,x)) (C 360)

Occurrence and processuals

OccursAt(x,y) $\equiv_{def}$ (SpanEntity(x) $\wedge$ ~TemporalRegion(x) $\wedge$~SpatiotemporalRegion(x) $\wedge\exists z$ (TemporalSlice(z,x) $\wedge$ AtTime(z,y)) (D 361)

Processual(x) $\equiv_{def}$ $\exists y$ OccursAt(x,y) (D 362)

Process(x) $\equiv_{def}$ Processual(x) $\wedge$ StronglyConnected(x) $\wedge$ $\forall y$ ((Part(x,y) $\wedge$ StronglyConnected(y)) $\rightarrow$ x=y) (D 363)

Event(x) $\equiv_{def}$ $\exists y$ (Processual(y) $\wedge$ TemporalSlice(x, y)) (D 364)

Event-BF(x) $\equiv_{def}$ $\exists y$ (Processual(y) $\wedge$ TemporalSlice(x, y) $\wedge$ BoundaryFor(x, y)) (D 365)

Temporal order (on instants)

$\forall x \forall y$ (Before(x,y) $\rightarrow$ (TemporalInstant(x) $\wedge$ TemporalInstant(y))) (A 366)

$\forall x$ ~Before(x,x) (A 367)

$\forall x \forall y$ (Before(x,y) $\rightarrow$ ~Before(y,x)) (A 368)

$\forall x \forall y \forall z$ ((Before(x,y) $\wedge$ Before(y,z)) $\rightarrow$ Before(x,z)) (A 369)

$\forall x \forall y$ ((TemporalInstant(x) $\wedge$ TemporalInstant(y)) $\rightarrow$ (Before(x,y) $\vee$ (x = y) $\vee$ Before(y,x))) (A 370)

Mereological and existential axiom

Temporal region

$\forall x \forall y$ ((TemporalRegion(x) $\wedge$ TemporalRegion(y)) $\rightarrow$ $\exists z$ sum(x,y,z)) (A 371)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ TemporalRegion(y) $\wedge$ TemporalRegion(z)) $\rightarrow$ TemporalRegion(x)) (A 372)

$\forall x \forall y$ ((TemporalRegion(x) $\wedge$ Part(y,x)) $\rightarrow$ TemporalRegion(y)) (A 373)

possible axiom

$\forall x \forall y$ ((TemporalRegion(x) $\wedge$ Part(x,y)) $\rightarrow$ TemporalRegion(y)) (A 374)

Spatiotemporal region

$\forall x \forall y$ ((SpatiotemporalRegion(x) $\wedge$ SpatiotemporalRegion(y)) $\rightarrow$ $\exists z$ Sum(x,y,z)) (A 375)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ SpatiotemporallRegion(y) $\wedge$ SpatiotemporallRegion(z)) $\rightarrow$ SpatiotemporallRegion(x)) (A 376)

$\forall x \forall y$ ((SpatiotemporalRegion(x) $\wedge$ Part(y,x))

$\rightarrow$ SpatiotemporalRegion(y)) (A 377)

$\forall x \forall y$ ((SpatiotemporalRegion(x) $\wedge$ Part(x,y))
$\rightarrow$ SpatiotemporalRegion(y)) (A 378)

Processual entities

$\forall x \forall y$ ((Processual(x) $\wedge$ Part(y,x)) $\rightarrow$ Processual(y)) (A 379)

$\forall x \forall y$ ((Processual(x) $\wedge$ Processual(y)) $\rightarrow \exists z$ Sum(x,y,z)) (A 380)

$\forall x \forall y$ (Sum(x,y,z) $\wedge$ Processual(y) $\wedge$ Processual(z))
$\rightarrow$ Processual(x)) (A 381)

$\forall x \forall y$ (Process(x) $\wedge$ Process(y) $\wedge$ p(x, y)) $\rightarrow$ x=y) (A 382)

possible axiom

$\forall x \forall y$ ((Processual(x) $\wedge$ Part(x,y)) $\rightarrow$ Processual(y)) (A 383)

Span entity

PT(SpanEntity,Processual,TemporalRegion,SpatiotemporalRegion) (A 384)
$\forall x$ (SpanEntity(x) $\leftrightarrow$ (Processual(x) $\vee$ TemporalRegion(x) $\vee$
SpatiotemporalRegion(x)) $\wedge$ ~(Processual(x) $\wedge$ TemporalRegion(x)) $\wedge$
~(Processual(x) $\wedge$ SpatiotemporalRegion(x)) $\wedge$ ~(TemporalRegion(x) $\wedge$
SpatiotemporalRegion (x)))) (A 134')

$\forall x \forall y$ ((SpanEntity(x) $\wedge$ Part(y,x)) $\rightarrow$ SpanEntity(y)) (A 385)

$\forall x \forall y$ ((SpanEntity(x) $\wedge$ Part(x,y) $\rightarrow$ SpanEntity(y)) (A 386)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ SpanEntity(y) $\wedge$ SpanEntity(z))
$\rightarrow$ SpanEntity(x)) (A 387)

$\forall x$ (SpanEntity(x) $\rightarrow \exists y$ TemporalPart(y,x)) (A 388)

$\forall x$ (SpanEntity(x) $\rightarrow \exists y$ SpatiotemporalPart(y,x)) (A 389)

$\forall x \forall y$ ((TemporalRegion(x) $\wedge$ SpatiotemporalPart(y,x)) $\rightarrow$ x=y) (A 390)

4.9 SNAP and SPAN

Material in this section is partially based on or adapted from (Grenon, 2003c; Grenon, 2003b; Grenon and Smith, 2003).

Primitive terms
The native primitive terms of BFO are non temporalized predicates for the relation of participation and realization which obtains at a given moment of time. ParticipatesIn(x,y) means that x participates in y (x exists at the time of obtainment of the relation and y is located at that time). The most fundamental form of participation is thus between a SNAP entity and a temporal slice of a process (an

event) – (Grenon, 2003c). It is this variant but with the mention of the time of location of the event in question which I will take as primitive. The other native primitive is Realization(x,y) where x is a trope – with analogous qualifications.

ParticipatesAt(x,y,z) means that x is a substantial or a trope which participates in event y at z.

RealizedAt(x,y,z) means that x is in a process of realization in the event y at z.

Defined terms

STParticipantAt(x,y,z) means that there is an instantaneous spatiotemporal part of y at z in which x participates.

TParticipantAt(x,y,z) means that there is a temporal slice of y at z in which x participates.

CParticipantIn(x,y) means that x is a complete participant in y, i.e., x participates in each temporal slice of y at the time at which it occurs.

Life(x,y) means that x is the life of y. The life of a substantial is the fusion of all processuals it is a complete participant of.

SpatialLocalizationAt(x,y,z) means that the event x is spatially localized at the spatial region y at the instant of time z; y is the spatial location of the fusion of the location of the participants in x at z.

Functioning(x) means that x is a functioning process, i.e., the realization of a function.

Axioms for ParticipatesAt

$\forall x \forall y \forall z$ (ParticipatesAt(x,y,z)
$\rightarrow$ (ExistsAt(x,z) $\wedge$ AtTime(y,z) $\wedge$ SDAt(y,x,z))) (A 391)

$\forall x \forall y \forall z$ (ParticipatesAt(x,y,z) $\rightarrow$
(Substantial(x) $\wedge$ Event(y) $\wedge$ TemporalInstant(y))) (C 392)

possible axioms

$\forall x$ (Processual(x) $\rightarrow \exists y \exists z$ ParticipatesAt(y,x,z)) (A 393)

$\forall x$ (Substantial(x) $\rightarrow \exists y \exists z$ ParticipatesAt(x,y,z)) (A 394)

Definitions

STParticipantAt(x,y,z) $\equiv_{def}$ $\exists w$ (SpatiotemporalPart(w,y)
$\wedge$ ParticipatesAt(x,w,z)) (A 395)

TParticipantAt(x,y,z) $\equiv_{def}$ $\exists w$ (TemporalPart(w,y)
$\wedge$ ParticipatesAt(x,w,z)) (A 396)

CParticipantIn(x,y) $\equiv_{def}$ $\forall z$ (OccursAt(y,z) $\rightarrow$ TParticipantAt(x,y,z)) (A 397)

Life(x,y) $\equiv_{def}$ Fusion(x,w[CParticipantIn(y,w)]) (A 398)

SpatialLocalizationAt(x,y,z) $\equiv_{def}$ Fusion(y,w[$\exists v$ (ParticipatesAt(v,x,z)

$\wedge$ SpatialLocationAt(v,w,z))]]) (A 399)

possible corollaries

$\forall$x (Substantial(x) $\rightarrow$ $\exists$y Life(y,x)) (A 400)

$\forall$x$\forall$y (Life(x,y) $\rightarrow$ SD(x,y)) (A 401)

possible axioms (can this at any rate characterize a kind of substantial entities?)

$\forall$x$\forall$y$\forall$z$\forall$w (ParticipatesAt(x,y,z) $\wedge$ (Part(w,x))
$\rightarrow$ ParticipatesAt(w,y,z)) (A 402)

Axioms for RealizedAt

$\forall$x$\forall$y$\forall$z (RealizedAt(x,y,z)
$\rightarrow$ $\exists$w (InheresIn(x,w,z) $\wedge$ ParticipatesAt(w,y,z)) (A 403)

$\forall$x$\forall$y$\forall$z ((InheresIn(x,y,z) $\wedge$ RealizedAt(x,w,z))
$\rightarrow$ ParticipatesAt(y,w,z))) (A 404)

$\forall$x$\forall$y$\forall$z$\forall$v$\forall$w (RealizedAt(x,y,z) $\wedge$ (InheresIn(x,v,z) $\wedge$ Life(w,z))
$\rightarrow$ Part(x,w,z)) (C 405)

Functioning(x) $\equiv_{def}$ $\exists$y$\exists$z (RealizedAt(x,y,z) $\wedge$ Function(y)) (A 406)

4.X SNAP

For the sake of completion, SNAP is given in its native (non temporalized version).

Material in this section is based on or adapted from (Grenon, 2003b; Grenon and Smith, 2003).

Primitive terms

The term space designates an individual, the spatial universe. It is an independent entity in the broad sense of this term.

SpatialLocation(x,y) means that the SNAP entity x is located at the spatial region y. (This is exact location.)

InheresIn(x,y) means that the trope x inheres in the substantial y. It is direct inherence. (Defining inherence brings about too much sophistications at this stage, in addition, some features of tropes of more specific kinds which are not yet clarified may conflict with the prospective definition.)

Defined terms

SpatialRegion(x) means that x is a spatial region, i.e., a part of space.

SpatialSubsumption(x,y) means that x spatially subsumes y, i.e., the spatial location of x is a part of the spatial location of y.

Substantial(x) means that x is a substantial entity, i.e., an independent SPAN entities which does not overlap with space (substantial entities are located in space).

Substance(x) means that x is a substance, i.e., it is a maximally strongly connected substantial entity. It has a bona fide boundary.

Occupies(x,y) means that x occupies y, i.e., i) x and y (which are both substantial entities) do not overlap and neither do their respective locations, but ii) the location of x is an internal part of the location of the sum of the x and y.

Site(x) means that x is a site, i.e., it is a substantial entity *occupied by* a substance.

Trope(x) means that x is a trope, i.e., it is a SNAP entity which specifically depends on at least one substantial entity, in addition, it does not overlaps with any spatial region (but it is located in space). A number of species of the category of tropes are mentioned, though not more formally characterized than taxonomically. Their theories are still work in progress – so I don't know what these are -- and prospectively requires a modal apparatus (unknown too).

MTrope(x) means that x is a monadic trop, i.e., it is specifically dependent on at most on one substantial entity.

RTrope(x) means that x is a relational trope, i.e., it is specifically dependent on at least two substantial entities.

Main subcategories of SNAP entities

$\mathrm{SpatialRegion}(x) \equiv_{def} \mathrm{Part}(x, \mathrm{space})$ (D 407)

$\mathrm{Substantial}(x) \equiv_{def} (\mathrm{SnapEntity}(x) \wedge \sim\exists y\, \mathrm{SD}(x,y) \wedge \sim\mathrm{SpatialRegion}(x))$ (D 408)

$\mathrm{Substance}(x) \equiv_{def} \mathrm{Substantial}(x) \wedge \mathrm{StronglyConnected}(x)$
$\wedge\ \forall y\, ((\mathrm{Part}(x,y) \wedge \mathrm{StronglyConnected}(y)) \rightarrow x=y)$ (D 409)
$\mathrm{Occupies}(x,y) \equiv_{def} (\mathrm{Substantial}(x) \wedge \mathrm{Substantial}(y) \wedge \sim\mathrm{Overlaps}(x,y)$
$\wedge\ \forall v \forall w \forall z\, ((\mathrm{SpatialLocation}(x,v) \wedge \mathrm{SpatialLocation}(y,w)$
$\wedge\ \mathrm{Sum}(z,v,w)) \rightarrow (\sim\mathrm{Overlaps}(v,w) \wedge \mathrm{InternalPart}(v,z)))$ (D 410)

$\mathrm{Site}(x) \equiv_{def} \exists y\, (\mathrm{Substance}(y) \wedge \mathrm{Occupies}(y,x))$ (D 411)
$\mathrm{Trope}(x) \equiv_{def} \exists y\, \mathrm{InheresIn}(x,y)$ (D 412)

$\mathrm{MTrope}(x) \equiv_{def} \forall y \forall z\, ((\mathrm{InheresIn}(x,y) \wedge \mathrm{InheresIn}(x,z)) \rightarrow x = z)$ (D 413)

$\mathrm{RTrope}(x) \equiv_{def} \exists y \exists z\, (\mathrm{InheresIn}(x,y) \wedge \mathrm{InheresIn}(x,z) \wedge \sim(y = z))$ (D 414)

$\mathrm{SpatialSubsumption}(x,y) \equiv_{def} \forall v \forall w\, ((\mathrm{SpatialLocation}(x,v)$
$\wedge\ \mathrm{SpatialLocation}(z,w)) \rightarrow \mathrm{Part}(w,v))$ (D 415)

Spatial location

$\forall x \forall y\, (\mathrm{SpatialLocation}(x,y) \rightarrow (\mathrm{SnapEntity}(x) \wedge \mathrm{SpatialRegion}(y)))$ (A 416)

$\forall x \forall y \forall z\, ((\mathrm{SpatialLocation}(x,y) \wedge \mathrm{SpatialLocation}(x,z)) \rightarrow x = z)$ (A 417)

$\forall x\, (\mathrm{SnapEntity}(x) \rightarrow \exists y\, \mathrm{SpatialLocation}(x,y))$ (A 418)

Inherence

$\forall x \forall y\, (\mathrm{InheresIn}(x,y) \rightarrow (\mathrm{SnapEntity}(x) \wedge \sim\mathrm{SpatialRegion}(x)$

$\wedge$ ~Substantial(x) $\wedge$ Substantial(y) $\wedge$ SD(x,y))) (A 419)

$\forall x \forall y$ ((InheresIn(x,y) $\wedge$ SpatialLocation(x,v)
$\wedge$ SpatialLocation(y,w))$\rightarrow$ v = w) (A 420)

There are no bare particulars.
$\forall x \forall y$ ((Substantial(x) $\rightarrow \exists y$ InheresIn(y,x)) (A 421)

Subcategories of that of tropes (some examples)

PT(Trope,MTrope,RTrope) (A 422)

$\forall x$ ((Trope(x) $\leftrightarrow$ (MTrope(x) $\vee$ RTrope(x))
$\wedge$ ~(Mtrope(x) $\wedge$ RTrope(x))) (A 61′)

SB(Trope,Function) (A 423)

$\forall x$ (Function(x) $\rightarrow$ Trope(x)) (A 62′)

SB(Trope,Quality) (A 424)

$\forall x$ (Quality(x) $\rightarrow$ Trope(x)) (A 63′)

SB(Trope,Role) (A 425)

$\forall x$ (Role(x) $\rightarrow$ Trope(x)) (A 64′)

DJ(Quality,Function) (A 426)

$\forall x$ ~(Quality(x) $\wedge$ Function(x)) (A 65′)

DJ(Role,Function) (A 427)

$\forall x$ ~(Role(x) $\wedge$ Function(x)) (A 66′)

DJ(Role,Quality) (A 428)

$\forall x$ ~(Role(x) $\wedge$ Quality(x)) (A 67′)

possible axiom

SB(RTrope,Role) (A 429)

$\forall x$ (Role(x) $\rightarrow$ RTrope(x)) (A 68′)

Mereological and existential axioms and corollaries

Substantial entities

$\forall x \forall y$ ((Substantial(x) $\wedge$ Substantial(y)) $\rightarrow \exists z$ sum(z,x,y)) (A 430)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ Substantial(y) $\wedge$ Substantial(z))
$\rightarrow$ Substantial(x)) (A 431)

$\forall x \forall y$ (Substantial(x) $\wedge$ Part(y,x) $\rightarrow$ Substantial(y)) (A 432)

Possible corollaries

$\forall x \forall y$ ((Substance(x) $\wedge$ Substance(y) $\wedge$ Part(x, y)) $\rightarrow$ x=y) (C 433)

$\forall x$ (Substantial(x) $\leftrightarrow$ $\exists y$ (Substance(y) $\wedge$ Overlaps(x,y))) (C 434)

possible axiom

$\forall x \forall y$ ((Substantial(x) $\wedge$ Part(x,y)) $\rightarrow$ Substantial(y)) (A 435)

$\forall x \forall y$ ((Substantial(x) $\wedge$ Substantial(x) $\wedge$ SpatialSubsumption(x,y))
$\rightarrow$ Part (y,x)) (A 436)

Tropes

$\forall x \forall y$ ((Trope(x) $\wedge$ Trope(y)) $\rightarrow$ $\exists z$ sum(z,x,y)) (A 437)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ Trope(y) $\wedge$ Trope(z)) $\rightarrow$ Trope(x)) (A 438)

$\forall x \forall y$ ((Trope(x) $\wedge$ Part(y,x)) $\rightarrow$ Trope(y)) (A 439)

possible axiom

$\forall x \forall y$ ((Trope(x) $\wedge$ Part(x,y)) $\rightarrow$ Trope(y)) (A 440)

Spatial regions

SnapEntity(space) (A 441)

$\forall x$ (SpatialRegion(x) $\rightarrow$ ~$\exists y$ SD(x,y)) (A 442)

~Substantial(space) (C 443)

$\forall x \forall y$ ((SpatialRegion(x) $\wedge$ SpatialRegion(y)) $\rightarrow$ $\exists z$ sum(z,x,y)) (A 444)

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ SpatialRegion(y) $\wedge$ SpatialRegion(z))
$\rightarrow$ SpatialRegion(x)) (A 445)

$\forall x \forall y$ ((SpatialRegion(x) $\wedge$ Part(y,x)) $\rightarrow$ SpatialRegion(y)) (A 446)

possible axiom

$\forall x \forall y$ ((SpatialRegion(x) $\wedge$ Part(x,y)) $\rightarrow$ SpatialRegion(y)) (A 447)

Snap entities

$\forall x \forall y \forall z$ ((Sum(x,y,z) $\wedge$ SnapEntity(y) $\wedge$ SnapEntity(z))
$\rightarrow$ SnapEntity(x)) (A 448)

$\forall x \forall y$ ((SnapEntity(x) $\wedge$ Part(x,y)) $\rightarrow$ SnapEntity(y)) (A 449)

PT(SnapEntity,Substantial,Trope,SpatialRegion) (A 450)

$\forall x$ (SnapEntity(x) $\leftrightarrow$ (Substantial(x) $\vee$ Trope(x) $\vee$ SpatialRegion(x))
$\wedge$ ~(Substantial(x) $\wedge$ Trope(x)) $\wedge$ ~(Substantial(x) $\wedge$ SpatialRegion(x))
$\wedge$ ~(SpatialRegion(x) $\wedge$ Trope(x)))) (A 89')

References

References for section 4

Casati, R. and Varzi, A. C. (1996). The Structure of Spatial Localization, *Philosophical Studies*, 82: 2, 205-239.

Casati, R. and Varzi, A. C. (1999). *Parts and Places: The Structures of Spatial Representation*. Cambridge, MA: The M.I.T. Press.

Grenon (2003b) Spatio-temporality in Basic Formal Ontology: SNAP and SPAN, Upper-Level Ontology, and Framework of Formalization (Part I), *IFOMIS Technical Report Series*, 05/2003.

Grenon, P. (2003c) Spatio-temporality in BFO: Trans-ontology (Part II), manuscript.

Grenon and Smith (2003) SNAP and SPAN: Towards Dynamic Geospatial Ontology, forthcoming in *Spatial Cognition and Computation*, 4:1 (2004).

Simons, P. (1987). *Parts: A Study in Ontology*. Oxford: Clarendon Press.

Smith, B. (1997). On Substances, Accidents and Universals: In Defence of a Constituent Ontology. *Philosophical Papers* 26, 105–127.

Smith, B. (1998) The Basic Tools of Formal Ontology, in Nicola Guarino (ed.), *Formal Ontology in Information Systems* Amsterdam, Oxford, Tokyo, Washington, DC: IOS Press (Frontiers in Artificial Intelligence and Applications), 19–28.

Smith, B. and Varzi, A. C. (2000). Fiat and Bona Fide Boundaries. *Philosophy and Phenomenological Research*, 60: 2, 401–420.